



Review

Comprehensive review of shear resistance and web stiffeners design in plate girders[☆]Sérgio Nascimento^{a,*}, Filip Ljubinković^b, João P. Martins^b, José J. Oliveira Pedro^c^a Structural Monitoring Unit, Structures Department, National Laboratory for Civil Engineering (LNEC), Av. Do Brasil 101, 1700-066 Lisboa, Portugal^b IRISE, ARISE, Department of Civil Engineering, University of Coimbra, Rua Luís Reis Santos – Pólo II, 3030-788 Coimbra, Portugal^c CERIS, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

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ABSTRACT

This paper presents a comprehensive review of the role of transverse, longitudinal, and diagonal web stiffeners in enhancing the shear buckling resistance of I-shaped steel plate girders. Over the past few decades, extensive laboratory experiments, analytical formulations, and numerical analyses have been conducted to better understand the shear behaviour of stiffened webs, supporting the development of more efficient and reliable design methods.

The review summarises and critically compares the main design models for elastic buckling and ultimate shear resistance, as well as the essential design requirements for web stiffeners. Key investigations are compiled in detailed tables, and existing analytical expressions are examined in terms of their assumptions, boundary conditions, and accuracy.

In addition, a finite element (FE) model is developed to evaluate the shear performance of girders with different stiffener configurations – including transverse, longitudinal, or diagonal – and its predictions are benchmarked against available design methods. The FE results complement the literature findings by highlighting general trends, verifying analytical formulations, and assessing the conservatism of current design rules.

Overall, the study demonstrates that while existing models predict the shear buckling resistance of stiffened plate girders reasonably well, notable gaps remain regarding the interaction between stiffeners, the treatment of diagonal stiffeners, and the influence of torsional rigidity. The review identifies these challenges and outlines future directions for improving both the accuracy and efficiency of web stiffener design.

1. Introduction

Plate buckling resistance (also referred to as ultimate resistance) and plate elastic critical buckling load of a steel plate are directly influenced by the support and loading conditions, as well as the material and geometric properties of the plate, including its slenderness ($\lambda_w = h_w/t_w$), web panel aspect ratio ($\alpha = a/h_w$), curvature (r), and taper ratio (ϕ). Nowadays, slender plates are typically used in webs of plate girders due to their optimisation potential. It is therefore important to consider their plate buckling behaviour under combined stresses (e.g., direct in-plane and shear) (Fig. 1) to mitigate conservative design assumptions that could lead to increased material consumption and fabrication costs [1].

In steel plate girders with flat webs, the shear buckling resistance can be enhanced by increasing the web thickness (t_w), thereby reducing the

slenderness of the panel. However, this approach is often uneconomical for long-span structures such as bridges. A more efficient alternative involves stiffening the web, as illustrated in Fig. 2, while maintaining or even reducing its thickness [2]. Different web stiffeners can be adopted, as illustrated in Fig. 3. These include the use of transverse stiffeners alone, or a combination of transverse stiffeners with either longitudinal or diagonal stiffeners. By stiffening the web, the panel dimensions are decreased, leading to an increase in both the panel elastic critical buckling stress (σ_{cr} and τ_{cr}) and post-buckling resistance (σ_u and τ_u). Nevertheless, this method raises important design considerations: What is the most effective configuration for the web stiffeners to enhance shear resistance? Which models can be used for evaluating the shear resistance of the girder? And what design rules should be adopted for the sizing and arrangement of web stiffeners? Over the past century,

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* Corresponding author.

E-mail address: snascimento@lnec.pt (S. Nascimento).

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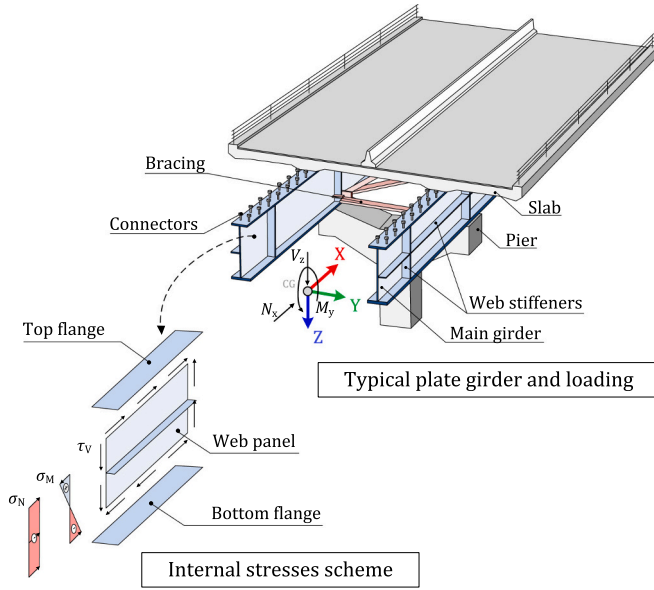


Fig. 1. Loading conditions on the web of a twin steel plate girder.

numerous researchers have sought to address these questions, proposing various models for the design of steel plate girders with webs stiffened by transverse, longitudinal, or diagonal stiffeners.

A web stiffener is characterized by the following non-dimensional key design parameters based on its geometry, as illustrated in Fig. 3. The index nomenclature considered is as follows: s – stiffener; t – transverse; l – longitudinal, and d – diagonal.

$$\text{Stiffener slenderness } \lambda_s = b_s/t_s \quad (1)$$

$$\text{Stiffener relative slenderness } \bar{\lambda}_s = \sqrt{f_{y,s}/\sigma_{cr,s}} \quad (2)$$

$$\text{Stiffener relative axial rigidity } \delta_s = \frac{A_s}{b t_w} \quad (3)$$

$$\text{Stiffener relative bending rigidity } \gamma_s = \frac{E I_s}{D b} \quad (4)$$

$$\text{Stiffener relative torsional rigidity } \theta_s = \frac{G J_s}{D b} \quad (5)$$

$$\text{Web bending rigidity } D = \frac{E t_w^3}{12 (1 - \nu^2)} \quad (6)$$

In these formulas, A_s , I_s , and J_s denote the stiffener's cross-sectional area, second moment of area, and torsional constant, respectively. The

second moment of area of the stiffener about the strong axis ($I_{s,y}$) and the weak axis ($I_{s,z}$) is calculated about the z-z and y-y axes, respectively, as indicated in Figs. 2 and 3. Material properties are defined by the modulus of elasticity E , shear modulus G , yield stress $f_{y,s}$, and the Poisson coefficient ν . The stiffener's elastic buckling load is $\sigma_{cr,s}$ and D denotes the bending rigidity of the web plate.

The width of a stiffened panel, b , is given by Eq. (7) and varies based on standards and researchers. For transverse stiffeners, Eq. (7) limits panel width to h_w which maintains a constant stiffener bending rigidity (γ_s) for long webs ($\alpha > 1$). This approach aligns with several authors [3] and actual standards such as AASHTO:2024 [4] and AISC 360:2020 [5]. However, recent studies [6] have investigated the stiffener requirements for long panels and support using the full panel width instead ($b=a$), arguing that otherwise the design can be overly conservative. Similarly, Kim et al. [7] suggested that stiffener requirements decrease in long panels, making width selection less critical. This view was previously endorsed by former AASHTO:2005 [12] and AISC 360:2005 [13].

$$b = \begin{cases} a & \text{for } \alpha < 1 \\ h_w & \text{for } \alpha \geq 1 \end{cases} \quad (7)$$

In the last 50 years, extensive research has focused on evaluating the true impact of stiffeners in steel plate girders' design. This paper provides a detailed review of the latest contributions to enhancing structural shear buckling resistance through stiffening. It also maintains a historical perspective on key works in this field. Specifically, the paper examines the design of webs of plate girders subjected to shear stresses and optimisation of web stiffener geometry. This includes positioning for transverse, longitudinal, and diagonal stiffeners (Fig. 2) and their cross-sectional shape mechanical properties (Fig. 3). Where appropriate, it also reviews existing design methodologies for the design of steel plate girders with stiffened webs. The aim is to offer a comprehensive synthesis of current knowledge and practical design approaches.

Design practice indicates that the failure mode associated with flange-induced buckling seldom governs the web slenderness. However, if the simplified expression of EN 1993-1-5:2006 [8] is applied to the design of plate girders in high-strength steel (HSS), it results in significantly lower slenderness limits, which may control the web design and restrict the potential for reducing the web thickness to take advantage of the higher steel strength [9]. Recent research has shown that this formulation is generally conservative [10], and consequently, improvements have been introduced in the new version of the standard EN 1993-1-5:2024 [11].

Hence, Section 2 provides a comprehensive review of the design of transversely stiffened webs, which significantly influences shear behaviour and enhances the shear buckling coefficient, k_τ . Section 3 examines advancements in web design, considering longitudinal stiffeners, which improve the web's resistance to shear and direct stresses. These stiffeners enhance k_τ and improve the web's effectiveness in the compressed zone, reducing direct stress redistribution to the

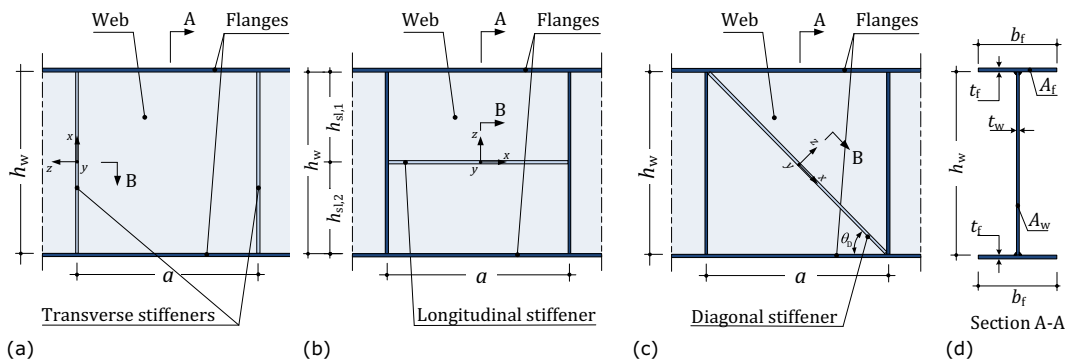


Fig. 2. Types of web stiffeners in plate girders depending on their alignment: (a) transverse stiffeners, (b) longitudinal stiffeners, (c) diagonal stiffeners, and (d) plate girder cross-section A-A.

compression flange. Section 4 reviews recent developments in the design of webs diagonally stiffened, which, like transverse stiffeners, primarily enhance web shear behaviour by increasing k_τ .

Finally, in Section 5, the main design methods for stiffened webs presented in the previous sections are compared with the results of numerical simulations, using several illustrative examples.

2. Transversely stiffened webs

2.1. Shear buckling resistance of transversely stiffened webs

This section examines the shear buckling behaviour of plate girders with transversely stiffened webs, outlining the main analytical models and findings that form the basis of current design provisions.

2.1.1. Research historical background

In 1886, Wilson [14] observed that a plate girder's web plate reaches a new equilibrium state after buckling, with additional reserve capacity beyond the web buckling load. He described the post-buckling behaviour as resembling the response of a Pratt truss. However, at the time, no formal method existed for evaluating the post-buckling resistance, and the design of webs in civil engineering structures was based solely on linear elastic theory [15].

The BS 153:1958 [16] standard for bridge design was the first to recognise the post-buckling behaviour of webs by introducing a load factor. Research on the ultimate shear resistance of web plates began nearly a century ago and has evolved alongside studies on the shear behaviour of unstiffened and stiffened slender web panels. To assess the post-buckling resistance of the web, several models have been developed to predict the ultimate shear resistance of webs. Notable among these is Basler's model (1960s) [17], the Cardiff theory by Porter et al. (1970s) [18], and Höglund's rotated stress field model (1970s) [19,20]. These models differ primarily in their assumptions regarding the rigidity of the web boundary members, such as the flanges and transverse stiffeners. Despite their differences, there is a consensus that the ultimate resistance of a steel plate girder under shear involves three primary mechanisms: (i) web pre-buckling (V_{cr}), (ii) web post-buckling (V_t), and (iii) plate girder collapse (V_m), given by Eq. (8). An essential literature review on the most relevant shear buckling resistance models may be consulted in Hasan et al. work [21].

$$V_u = V_{cr} + V_t + V_m \quad (8)$$

Basler's work led to the inclusion of post-buckling resistance requirements in standards such as AISC 1963 [22], AASHTO:1973 [23], SIA 161:1979 [26] and in BS 5400:1982 [27]. It remains the foundation of SIA 263:2003 [28], AISC 360:2022 [5] and AASHTO:2024 [4]. A modified version of Porter et al. proposal was adopted in BS 5400:1982 [27] and ENV 1993-1-1:1992 [29]. The rotating stress field model was adopted by EN 1993-1-5:2006 [8] and still used in 2024's version [11], as well as in AISC 360:2022 [5] for unstiffened webs.

2.1.2. Elastic critical buckling

The use of rigid transverse web stiffeners has a clear benefit on the shear elastic buckling behaviour of web plates. The linear elastic theory

[15] is commonly employed to calculate the elastic shear stress of the web that buckles (Eq. (9)), which is influenced by two key factors: the web slenderness (λ_w) and the web panel aspect ratio (α), together with elastic material properties (E and ν). As these geometrical parameters increase, the web's buckling resistance decreases. For long panels with an aspect ratio of $\alpha = 3$, the influence of stiffening becomes marginal.

While it is well established that the flanges and web stiffeners provide some rotational rigidity to the web plate [30,31,32], the shear buckling coefficient (k_τ) in Eq. (10) is calculated for simply supported rectangular webs, as proposed by Timoshenko [15] and adopted in EN 1993-1-5:2024 [11] and AASHTO:2024 [4].

$$\tau_{cr} = k_\tau \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t_w}{h_w} \right)^2 \quad (9)$$

$$k_\tau = \begin{cases} 4.00 + \frac{5.34}{\alpha^2}, & \text{for } \alpha \leq 1 \\ 5.34 + \frac{4.00}{\alpha^2}, & \text{for } \alpha > 1 \end{cases} \quad (10)$$

2.1.3. Ultimate resistance

Höglund's original rotated stress field model [19] underwent empirical modifications to account for transverse and longitudinal stiffeners, flange contributions to shear resistance, and the influence of potential geometric imperfections. These adjustments led to a revised model [20], which is present today in the design rule of EN 1993-1-5:2024 [11]. This model does not explicitly identify the three resisting mechanisms described; conversely, the buckling shear resistance (V_b) is calculated assuming the web ($V_{b,w}$) and the flanges ($V_{b,f}$) contributions as in Eq. (11), but limited by the modified web's shear plastic resistance (η is the hardening factor and is 1.2 for steel grades up to S460 and 1.0 for higher steel grades). The web resistance is estimated using Eq. (12) and the shear buckling reduction factor (χ_w) is given in Eq. (13) for intermediate panels and end panels with rigid end post, where the relative web slenderness is given in Eq. (14), while the shear buckling coefficient (k_τ) is defined according to Eq. (10). The collapse occurs due to the formation of four plastic hinges in the flanges after the yielding of the tension band of the web. Thus, the flange contribution proposed by Höglund's (in Eq. (15)) depends on the distance between plastic hinges formed at the collapse and is derived from the flange with the lowest axial resistance, and may be reduced if the flange is utilised.

$$V_u = V_b = V_{b,w} + V_{b,f} \leq \eta V_{pl} = \eta f_{y,w} h_w t_w / \sqrt{3} \quad (11)$$

$$V_{b,w} = \tau_u h_w t_w = \chi_w h_w t_w f_{y,w} / \sqrt{3} \quad (12)$$

$$\chi_w = \begin{cases} \eta & \text{for } \bar{\lambda}_w \leq 0.83/\eta \\ 0.83/\bar{\lambda}_w & \text{for } 0.83/\eta < \bar{\lambda}_w \leq 1.08 \\ 1.37/(0.7 + \bar{\lambda}_w) & \text{for } \bar{\lambda}_w > 1.08 \end{cases} \quad (13)$$

$$\bar{\lambda}_w = \frac{h_w}{37.4 t_w \varepsilon \sqrt{k_\tau}} \quad (14)$$

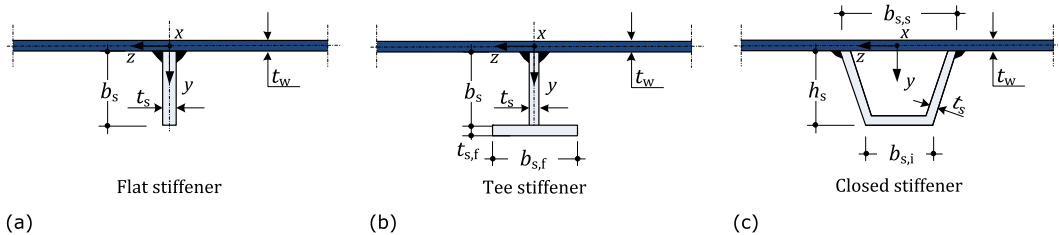


Fig. 3. Examples of web stiffeners in plate girders depending on their cross-section B-B: (a) flat stiffener, (b) tee stiffener, (c) closed stiffener.

$$V_{b,f} = \frac{b_f t_f^2 f_{y,f}}{c} \left(1 - \left(\frac{M_{Ed}}{M_{f,Rk}} \right)^2 \right) \text{ where } c = a \left(0.25 + 1.6 \frac{f_{y,f} b_f t_f^2}{f_{y,w} h_w^2 t_w} \right) \quad (15)$$

Currently, the contribution of the flanges has attracted the attention of several researchers [30,31,32], who have identified discrepancies between numerical results and the flange contribution as defined in EN 1993-1-5:2024 [11]. In particular, Pedro et al. observed that the c/a factor should not be treated as linear, and proposed an alternative formulation calibrated against both experimental and numerical results [33]. By adjusting the c/a factor, a significantly improved agreement was achieved between the shear resistance model of EN 1993-1-5:2024 and numerical results across a wide range of steel plate girder geometries [34]. This modification improved prediction accuracy and reduced non-conservativeness, namely for plate girders with high flange-to-web section modulus ratios, while reducing the scatter of the results.

2.1.4. Comparison between design models

The use of transverse web stiffeners has been shown to increase the ultimate shear buckling resistance (V_u). Over the years, numerous studies have proposed various models to evaluate the shear buckling resistance of steel plate girders. In [33] three key code-based models were compared: the model by Basler, adopted in SIA 263:2013 [28]); the model by Porter et al., initially developed as the tension field method and later incorporated into ENV 1993-1-1:1992 [29]); and Höglund's rotated stress field model, currently implemented in EN 1993-1-5:2024 [11].

Fig. 4 provides a graphical comparison of the shear buckling resistance of stiffened intermediate web panels calculated using these methods, for aspect ratios $\alpha = 1$ and 3. The reference geometry is defined by $h_w = 1000$ mm; $A_f/A_w = 1.0$; $b_f = 20 t_f$, and yield strengths $f_{y,w} = f_{y,f} = 355$ MPa.

2.2. Design of transverse stiffeners

The web of a welded plate girder, being slender and prone to local buckling, requires transverse stiffeners to resist shear forces and out-of-plane deformation (Fig. 2 (a)). These stiffeners, which may be flat plates, tee sections, or closed shapes (Fig. 3), enhance web stability by subdividing it into smaller panels, thereby increasing its elastic buckling resistance. While modern plate girders often carry shear loads beyond this elastic limit, stiffeners also contribute to post-buckling resistance mechanism. Additionally, they prevent flange inward movement – introducing axial forces – and may support bracing, act as bearing stiffeners, or resist support reactions during transport and erection.

2.2.1. Research historical background

The concept of transverse stiffener bending rigidity (γ_{st}) around the strong axis is traced back to 1849 when Fairbairn's experiments on tubular bridges highlighted the importance of web stiffeners. He observed that even a slight increase in metal significantly enhanced the structure's ultimate resistance [35]. In 1898, Wilson [14] expanded on this concept by investigating the effect of stiffener spacing on the shear buckling resistance [36]. By 1910, the American Railway Engineering Association's Bridge Specifications [37] introduced minimum dimensions for intermediate stiffeners, although they did not account for the spacing of transverse stiffeners. With the advent of welded construction, there was a renewed focus on refining plate girder design, particularly the optimisation of web plates and transverse stiffeners. Table 1 summarises key advancements in both elastic and ultimate design principles.

Timoshenko's *Theory of Elastic Stability* (of 1936) [15] established the $\gamma_{st,1}$ required for ribs and plate web stiffeners to achieve elastic buckling. That same year, Way [38] analysed plate stability under shear and bending, considering the effect of γ_{st} on elastic web buckling. In 1940, Moisseff [39] proposed a minimum stiffener bending rigidity based on Chwalla's panel design work [40]. Moore et al. [41] conducted experiments in 1942 on transverse stiffeners in plate girders, leading to an empirical design method. Kerensky et al. [42] later refined minimum second moment of area values for stiffeners. Stein and Fralich [43] (1949) conducted an experimental study, showing that increased bending rigidity raised the shear buckling coefficient (k_τ), though effects plateaued beyond a certain γ_{st} . Bleich [44] (1952) proposed an optimal γ_{st} for infinitely long stiffened plates. In 1956, Rockey [45] tested 220 aluminium web-stiffener combinations, deriving $\gamma_{st,1}$ requirements for the design of single- and double-sided stiffeners. Klöppel et al. [46] (1961) developed rigidity charts, still reliant on elastic critical load criteria. Skaloud et al. [47] (1967) found Moore et al.'s values underestimated requirements, recommending a threefold increase. Building on theoretical and experimental work, Massonnet et al. [48] proposed an optimal γ_{st} , while Huang [49] (1968) reviewed historical stiffener design research. More recently, Stanway et al. [50] (1993) examined γ_{st} effects on critical buckling loads in simply supported plates. Lee et al. [5] (2014) updated formulas for the minimum second moment of area needed to keep transverse stiffeners straight under critical web shear loading.

As post-buckling shear resistance was incorporated into standards in the late 20th century, transverse stiffener design rules evolved. Basler [51,17] proposed a formula for axial force (N_{st}), later accompanied by

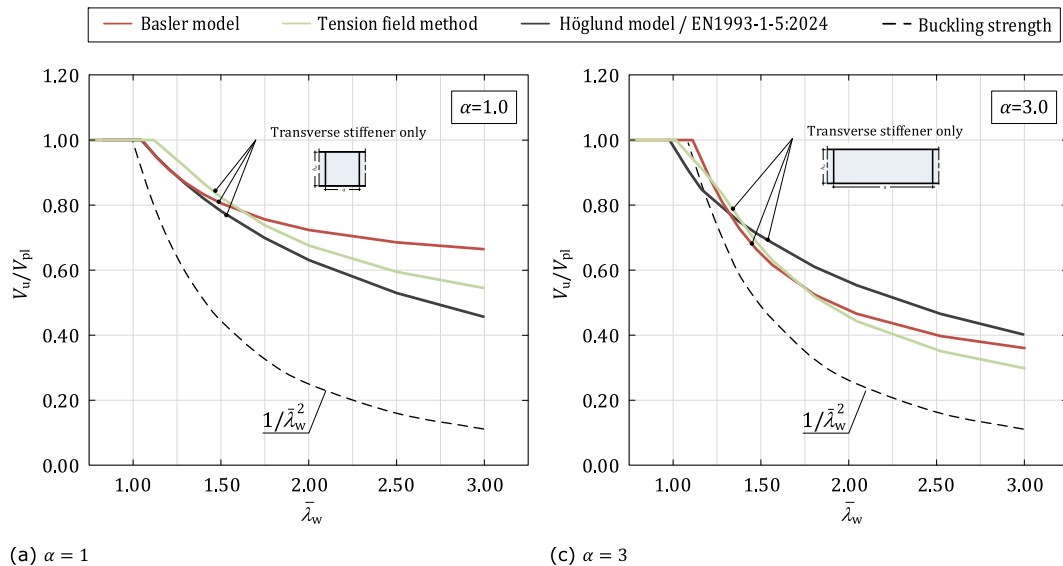


Fig. 4. Evolution of the relative shear buckling resistance of transversely stiffened webs V_u/V_{pl} vs. the relative web slenderness for aspect ratios: (a) $\alpha = 1$, (b) $\alpha = 3$.

Table 1

Overview of the investigations on transverse stiffeners reported in the literature.

Author	Year	Design	Load	Experimental		Numerical	Design proposal	
					Tests			Variable
Timoshenko [15]	1936	E	S	No	-	No	Yes	$\gamma_{st\ 1}$
Way [38]	1936	E	S; B	No	-	No	Yes	$\gamma_{st\ 1}$
Chwalla and Novak [40]	1937	E	S	No	-	No	Yes	$\gamma_{st\ 1}$
Moisseiff [39]	1940	E	S	No	-	No	Yes	$\gamma_{st\ 1}$
Moore et al. [41]	1942	E	S	Yes	60	No	Yes	$\gamma_{st\ 1}$
Stein and Fralich [43]	1949	E	S	Yes	20	No	No	-
Bleich [44]	1952	E	S	No	-	No	Yes	$\gamma_{st\ 1}$
Kerensky et al. [42]	1956	E	S	No	-	No	Yes	$\gamma_{st\ 1}$
Rockey [45]	1956	E	S	Yes	220	No	Yes	$\gamma_{st\ 1}$
Klöppel et al. [46]	1961	E	S	No	-	No	Yes	$\gamma_{st\ 1}$
Basler [17,51]	1961	U	S; B	Yes	13	No	Yes	N_{st}
Škaloud et al. [47]	1967	E	S	No	-	No	Yes	$\gamma_{st\ 1}$
Massonnet et al. [48,66,78]	1968	E	S	Yes	4	No	Yes	$\gamma_{st\ 1}$
Richmond [79]	1972	U	S	No	-	Yes	Yes	N_{st}
Porter et al. [18,80]	1975	U	S	Yes	24	No	Yes	N_{st}
Rockey et al. [52,80]	1981	U	S	Yes	24	No	Yes	N_{st}
Škaloud et al. [69]	1983	U	S	Yes	10	No	Yes	$\gamma_{st\ 2}$
Horne and Grayson [67,68]	1983	U	S	Yes	30	Yes	Yes	$\gamma_{st\ 2}$
Tang and Evans [53]	1984	U	S	Yes	9	No	No	-
Rahal [70]	1989	U	S	No	-	Yes	Yes	$\sigma_{st,max}$
Rahal and Harding [71,72,73]	1991	U	S	No	-	Yes	Yes	
Stanway et al. [3,50,55]	1996	U	S	No	-	Yes	Yes	$\omega_{st,lim}$
Mijušković et al. [74]	1999	U	S	No	-	No	Yes	$\gamma_{st\ 2}$
Xie [82]	2000	U	S; B	Yes	4	Yes	Yes	N_{st}
Lee et al. [57,81,56]	2003	E; U	S	Yes	6	Yes	Yes	$\gamma_{st\ 2}$
Alinia [76]	2005	E	S	No	-	Yes	No	-
Sinur et al. [65,77,83]	2013	U	S	Yes	6	Yes	Yes	$\gamma_{st\ 2}$
Kim et al. [7,58,75]	2014	U	S	No	-	Yes	Yes	$\gamma_{st\ 2}$
Lee et al. [5]	2014	E	S	No	-	Yes	Yes	$\gamma_{st\ 1}$
Xiao et al. [84]	2019	U	S	No	-	Yes	Yes	$\gamma_{st\ 2}$
Nascimento et al. [89,90,91]	2025	U	S	Yes	10	Yes	Yes	$\gamma_{st\ 2}$

Note: E stands for elastic web buckling load design, U for ultimate web load design, S for girder shear load, B for girder overall bending.

Porter et al. [18] and Rockey et al. [52] and Tang and Evans [53].

In 1980, Bjorhovde [54] found existing stiffener design methods overly conservative, particularly regarding area requirements, and called for more “*rational design rules*”. Moreover, Stanway et al. [3,55,50] (1990s) argued that out-of-plane forces, rather than axial forces, were more relevant for design. FE studies increasingly questioned stiffener area criteria, deeming them conservative and based on questionable assumptions [56]. Lee et al. [56] (2003) concluded that transverse stiffeners need not equilibrate compressive forces to sustain the diagonal tension field during post-buckling. Instead, they required only sufficient bending rigidity to maintain elastic shear buckling resistance. Many studies have since shown that axial forces in stiffeners are overestimated in design standards [52] [53,57] [58,59,60] [61] [62] [63] [64,65].

Research then shifted towards defining the stiffener rigidity ($\gamma_{st\ 2}$) needed to ensure effectiveness up to the ultimate girder capacity. Massonnet [66] (1954) was the first to highlight the need for 20 times increase in the existing elastic bending rigidity requirements. Grayson [67] (1981) later defined $\gamma_{st\ 2}$ to minimise its impact on load capacity. Horne and Grayson [68] (1983) conducted a parametric FE study, proposing a $\gamma_{st\ 2}$ ensuring stiffener functionality up to the ultimate girder capacity. That same year, Škaloud et al. [69] introduced a $\gamma_{st\ 2}$ formulation adjusting Massonnet’s $\gamma_{st\ 1}$ with a factor dependent on λ_w . Subsequent studies by Rahal [70] and Harding [71,72,73] recognised the role of out-of-plane forces in stiffener design but produced limited applications. Stanway et al. [3,55,50] (1990s) observed that stiffeners primarily resisted bending moments due to web deflection restraint and proposed a deflection limit. Mijušković et al. [74] (1999) used an energy-based approach to examine post-buckling stiffener rigidity. In 2014, Lee et al. [5] proposed a $\gamma_{st\ 2}$ formula and argued that area design

requirements were unnecessary. Kim et al. [75,58] (early 21st century) used FE-analysis to determine the $\gamma_{st\ 2}$ for ultimate shear resistance. More recently, Alinia [76] studied transverse stiffener rigidity, while Sinur et al. [77] (2013) proposed a calibrated design formula.

2.2.2. Elastic critical buckling

The elastic critical shear buckling of transversely stiffened plate girders marks the onset of web instability and is governed by the web panel’s aspect ratio, boundary conditions, and stiffener rigidity. This section reviews the main analytical formulations and design models used to design transverse web stiffener under elastic critical shear buckling loading.

2.2.2.1. Stiffener bending rigidity. With the increasing need for more precise methods in aircraft industry design in 1942, Moore et al. [41] conducted research on the effectiveness of transverse stiffeners in plate girders and proposed an empirical design method based on a rigidity criterion. By the early 1950s, the British Standards Institution Committee tasked Kerensky with developing rules for shear stresses in plate girder webs, as part of the BS 153 of 1958 revision (later renamed BS 5400-3). Based on Moore et al.’s studies, Kerensky et al. [42] proposed slightly revised values for the minimum second moment of area for stiffeners, setting its maximum permissible spacing at 1.5 times the web height (h_w), as described in Eq. (16). Kerensky’s proposals were later adopted in ENV 1993-1-1:1992 [29] and are still maintained in the current EN 1993-1-5:2024 [11], with α limit revised from 1.5 to $\sqrt{2} \approx 1.414$ as in Eq. (17).

$$I_{st\ 1} = 1.5 \frac{h_w^3 t_w^3}{a^2} \text{ for } \alpha < 1.5 \Leftrightarrow \gamma_{st\ 1} = 16.38 \frac{h_w^3}{a^2 b} \text{ for } \alpha < 1.5 \quad (16)$$

$$I_{st1} \geq \begin{cases} 1.5 \frac{h_w^3 t_w^3}{a^2} & \text{for } \alpha < \sqrt{2} \\ 0.75 h_w t_w^3 & \text{for } \alpha \geq \sqrt{2} \end{cases} \Leftrightarrow \gamma_{st1} \geq \begin{cases} 16.38 \frac{h_w^3}{a^2 b} & \text{for } \alpha < \sqrt{2} \\ 5.46 \frac{h_w^3}{b} & \text{for } \alpha \geq \sqrt{2} \end{cases} \quad (17)$$

In 1952, Bleich [44] proposed an optimal stiffener bending rigidity, γ_{st1} , for infinitely long stiffened plates, as described in Eq. (18). With a slight modification, Bleich's approach formed the basis for the design checks using AASHTO:2024 [4] and AISC 360:2022 [5], which was defined by the stiffener's second moment of area, I_{st1} , as shown in Eq. (19).

$$I_{st1} = \left(2.56 \frac{h_w^2}{a^2} - 1.83 \right) b t_w^3 \Leftrightarrow \gamma_{st1} = 28 \frac{h_w^2}{a^2} - 20 \quad (18)$$

$$I_{st1} = J b t_w^3 = \left(2.5 \frac{h_w^2}{a^2} - 2.0 \right) b t_w^3 \Leftrightarrow \gamma_{st1} = 27.3 \frac{h_w^2}{a^2} - 21.84 \quad (19)$$

2.2.2.2. Stiffener torsional rigidity. For stiffeners with open sections, unless a more advanced method of analysis is conducted to prevent their torsional buckling, the ratio between the Saint Venant torsional constant (I_T) of the isolated transverse stiffener and the polar second moment of area (I_p) at the point of attachment to the web must exceed $5.3 f_{y,st}/E$ [11]. This criterion aligns with limiting the stiffener's relative slenderness ($\bar{\lambda}_{st}$ in Eq. (2)) to $\bar{\lambda}_{st} \leq 0.70$ for stiffeners with minimal or no warping resistance. This corresponds to a critical elastic torsional buckling stress ($\sigma_{cr,st}$) of the transverse stiffener, calculated without considering the web plate's rotational restraint, exceeding $\sigma_{cr,st} \geq 2.04 f_{y,st}$. For a stiffener with warping rigidity, $\sigma_{cr,st}$ should exceed $6.25 f_{y,st}$, corresponding to a relative slenderness limit of $\bar{\lambda}_{st} \leq 0.40$.

2.2.3. Ultimate resistance

After the elastic buckling of the web, the transverse stiffeners allow the mobilisation of the web's tension field, enabling it to reach its ultimate shear resistance. This section presents the main analytical and design approaches used to evaluate the ultimate strength of transverse stiffeners in web plates.

2.2.3.1. Stiffener resistance. In the second half of the 20th century, the post-buckling resistance of the web plate was incorporated into design standards. Notable examples include AISC:1963 [22], AASHTO:1973 [23], SIA 161:1979 [26], BS 5400:1982 [27], and ENV 1993-1-1:1992 [29]. Subsequently, researchers started investigating the required area of transverse stiffeners, A_{st} , to resist compressive forces induced by diagonal tension in the post-buckling range. This requirement was integrated with the bending rigidity requirement, addressed in Sec. 2.1, to ensure web panels maintain adequate support at their boundary conditions after buckling occurs [57].

Basler's experimental tests from the 1960s [51,17] used strain gauges to assess the transverse stiffener's compression, N_{st} . He proposed a formula for this axial force when the maximum shear force on the girder is reached, as shown in Eq. (20), where the yield shear stress is $\tau_y = f_{y,w}/\sqrt{3}$ and $f_{y,w}$ is the web yield strength. AISC:1963 [22] and AASHTO:1973 [23] recognised that stiffeners should resist axial forces and recommended a minimum area (A_{st}) based on Basler's model [17]. However, this rule was omitted in the AASHTO:2007 [24] and AISC:2010 [25] editions, as a verification of stiffeners based solely on a bending rigidity (γ_{st}) criterion was adopted and deemed sufficient to ensure the pre- and post-buckling resistance of plate girders. The SIA 161 [26] also adopted Basler's rule, which remains in the 2013 SIA 263 edition [28].

$$N_{st} = \left(1 - \frac{\tau_{cr}}{\tau_y} \right) \times \frac{\alpha}{2} \times \left(1 - \frac{\alpha}{\sqrt{1 + \alpha^2}} \right) h_w t_w f_{y,w} \quad (20)$$

In 1975, Porter et al. [18] proposed that the stiffener axial forces should be treated like forces in an N-type truss (Pratt truss). Rockey et al. [52] developed a more complex model in 1981, based on shear buckling failure mechanisms, verified by a test campaign. They also proposed a simplified verification, assuming constant compression on the stiffener. In 1984, Tang and Evans [53] evaluated this compressive force and deemed Rockey et al.'s proposal "conservative", as it assumed all shear load after the web elastic buckling load, V_{cr} , was resisted by the stiffener:

$$N_{st} = V_{Ed} - V_{cr} \quad (21)$$

This design method primarily addresses in-plane forces from tension field action and does not account for out-of-plane forces from panel bending. These designs tend to be more conservative as the panel aspect ratio α decreases, as they assume transverse stiffeners resist high axial forces [82]. Rockey et al.'s axial force design was adopted in BS 5400 Part 3 in 1982 [27] and retained in the 2000 edition [85]. The same design approach was adopted in ENV 1993-1-1:1992 [29], later replaced by the EN 1993-1-5:2006 [8], and remains in the EN 1993-1-5:2024 edition [11]. This standard specifies that the ultimate resistance of the effective stiffener must be checked using Eq. (22) from EN 1993-1-1 [86], considering the design buckling curve c and buckling length $l \geq 0.75 h_w$. For asymmetric stiffeners, the bending moment M_{st} is calculated by accounting for the initial eccentricity between the stiffener's centre of gravity and the web, e (see Fig. 5), along with a sinusoidal initial imperfection $w_0 = s/200$, where s is the smallest of the adjacent web panel's dimensions, a_1 , a_2 or b :

$$\frac{N_{st}}{\chi_y N_{Rk}/\gamma_{M1}} + k_{yy} \frac{M_{st}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} \leq 1.0 \quad (22)$$

This design criterion for intermediate transverse stiffeners has been increasingly debated in recent years. According to various studies [3,7,50,56,58,59,87,88,100,101] the compression forces developed in the transverse stiffeners are significantly lower than the design force prescribed by Eq. (21). In fact, Hendy and Presta [87] demonstrated numerically that, for slender symmetrical steel plate girders as well as steel-concrete composite plate girders, the cross-sectional area of the stiffeners had no influence on the ultimate shear resistance. This conclusion was reached by substituting the actual transverse properties in the finite element models with transverse springs that provided only rigid out-of-plane restraint. A comprehensive discussion of the role of intermediate transverse stiffeners is presented in [33]. The design of transverse stiffeners has been recently reassessed [89,90,91], showing that it should primarily be governed by a stiffness criterion. Transverse stiffeners mainly contribute to the web's out-of-plane rigidity and are therefore predominantly subjected to local bending effects.

2.2.3.2. Stiffener bending rigidity. Later, in 2002, Lee et al. [57,56] determined that, for webs with $V_{cr}/V_{pl} < 0.8$, six times the transverse stiffener flexural rigidity specified by AASHTO:2005 [12] was sufficient, and suggested that under such conditions, area design requirements were unnecessary (where V_{pl} is the plastic web shear force). In 2004, Kim [75] conducted a FE analysis to determine the ultimate bending rigidity of stiffeners in I-shaped plate girders required to achieve ultimate shear resistance. Later, in 2007, Kim et al. [58] numerically defined the required stiffener rigidity I_{st2} , assuming that this level of stiffener bending rigidity would result in less than a 3% increase in the ultimate shear resistance of the plate girder. In 2014 [7], a simplified formulation of I_{st2} was introduced, by replacing the web thickness t_w from Eq. (23) and $b = \min \{a; h_w\}$ into Eq. (19). Solving this equation yields the required second moment of area I_{st2} for the plate girder to achieve its ultimate shear buckling load, as shown in Eq. (24).

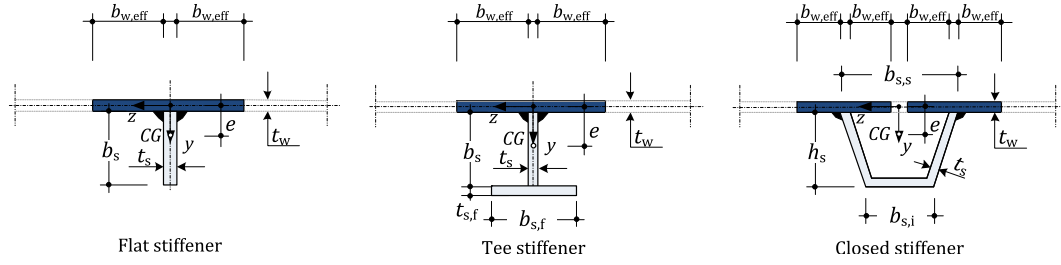


Fig. 5. Effective web stiffeners cross-section in plate girders depending on their cross-section: (a) flat stiffener, (b) tee stiffener, (c) closed stiffener.

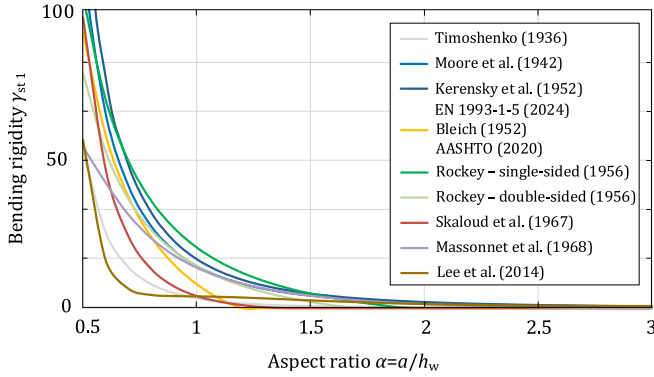


Fig. 6. Minimum elastic bending rigidity $\gamma_{st,1}$ of a transverse stiffener vs. aspect ratio according to each researcher and relevant design standards.

$$\frac{h_w}{t_w} = 1.10 \sqrt{\frac{E k_\tau}{f_{y,w}}} \Leftrightarrow t_w = \frac{h_w}{1.10} \sqrt{\frac{f_{y,w}}{E k_\tau}} \quad (23)$$

$$I_{st,2} \geq \frac{h_w^4}{\beta} \left(\frac{f_{y,w}}{E} \right)^{1.5} \Leftrightarrow \gamma_{st,2} \geq \frac{10.92}{\beta} \frac{h_w}{b} \lambda_w^3 \left(\frac{f_{y,w}}{E} \right)^{1.5} \quad (24)$$

The coefficient β is given in Eq. (25) but a constant value of 40 was defined by Kim et al. as a suitable value [7]. The possibility of considering transverse stiffeners with yield strength smaller than that of the web is addressed by $\rho_t = \max\{f_{y,w}/f_{y,st}; 1.0\}$. Hence, Kim et al.'s stiffener rigidity proposal of Eq. (26) was adopted by AASHTO:2007 [24] and AISC 360:2010 [25].

$$\beta = \frac{1.33 k_\tau^{1.5}}{\max\left\{\frac{2.5}{\alpha^2} - 2.0; 0.5\right\} \times \min\{\alpha; 1.0\}} \rightarrow \min\{\beta\} \approx 40 \quad (25)$$

$$I_{st,2} \geq \frac{h_w^4 \rho_t^{1.3}}{40} \left(\frac{f_{y,w}}{E} \right)^{1.5} \Leftrightarrow \gamma_{st,2} \geq 0.273 \frac{h_w}{b} \lambda_w^3 \rho_t^{1.3} \left(\frac{f_{y,w}}{E} \right)^{1.5} \quad (26)$$

Xiao et al. [84] used FE analysis to study stiffeners of high-strength steel girders and found that the AASHTO:2024 $I_{st,2}$ equation underestimates the stiffener requirement, proposing revised formulas.

As presented in Table 1, recent studies [89,90,91] have shown that the EN 1993-1-5 requirement for $I_{st,1}$ should be increased for the web to develop its full post-buckling resistance. This requirement is influenced by the size of the flanges.

2.2.4. Effective stiffener cross section

Many researchers concluded that the actual flexural rigidity of a welded stiffener in a real plate structure considerably increases compared to an isolated outstand stiffener, referring to a total effective web width of 18 to 45 t_w [40,44,52,49,78]. According to EN 1993-1-5:2024 [11], the effective cross-section of an intermediate transverse stiffener consists of the gross area of the stiffener plus the effective width

of the web, calculated as $b_{w,eff} = 15 \varepsilon t_w$ on each side of the transverse stiffener as in Fig. 5, where $\varepsilon = \sqrt{235/f_{y,w}}$. Conversely, AASHTO:2024 [4], AISC 360:2022 [5] and SIA 263 [28] specify that only the cross-sectional area of the transverse stiffener alone should be considered, and for the calculation of the second moment of area, the reference axis is taken along the edge in contact with the web for single-sided stiffeners and at the mid-thickness of the web for double-sided stiffeners, disregarding the contribution of any effective part of the web.

2.2.5. Comparison between design models

Fig. 6 compares the key proposals for the bending rigidity, $\gamma_{st,1}$, of transverse stiffeners designed for elastic web behaviour. The horizontal axis represents the aspect ratio, α , since no other geometric or material parameters influence the elastic shear behaviour expressed by the shear buckling coefficient, k_τ , in Eq. (10). The current EN 1993-1-5:2024 [11] adopts the minimum bending rigidity based on the proposal by Kerensky et al. [42], which was derived from the research by Moore et al. [41]. In contrast, the American standards [4,5] use the Bleich formulation [44] for determining the minimum elastic bending rigidity.

Fig. 7 (a), (b), and (c) further compare the main proposals for the minimum ultimate bending rigidity, $\gamma_{st,2}$, of transverse stiffeners under different web slenderness values ($\lambda_w = 180$, $\lambda_w = 120$, and $\lambda_w = 80$ respectively). To ensure consistency with Fig. 6, the horizontal axis in these figures also represents the aspect ratio, α . Both the current AASHTO:2024 [4] and AISC 360:2022 [5] standards adopt the proposal of Kim et al. [7] for designing transverse stiffeners to withstand the web panel's ultimate post-buckling resistance.

From these comparisons, it can be observed that the ultimate stiffener bending rigidity $\gamma_{st,2}$ is always more demanding than the elastic stiffener bending rigidity $\gamma_{st,1}$. Some proposals for $\gamma_{st,2}$ show dependence on the web slenderness λ_w and/or the steel grade $f_{y,w}$. In general, $\gamma_{st,2}$ increases as the web panel aspect ratio decreases. Moreover, higher web slenderness and stronger steel grades further raise the required ultimate bending rigidity.

3. Longitudinally stiffened webs

3.1. Shear buckling resistance of longitudinally stiffened webs

This section provides an overview of the governing mechanisms and analytical formulations describing the shear buckling resistance of longitudinally stiffened webs, together with a discussion of relevant experimental and numerical studies.

3.1.1. Research historical background

A longitudinally stiffened plate may experience either global or local buckling, with the lower elastic buckling load determining its behaviour:

- Global buckling: occurs when the stiffener and web deform together at the elastic buckling load, offering little or no enhancement to buckling resistance.

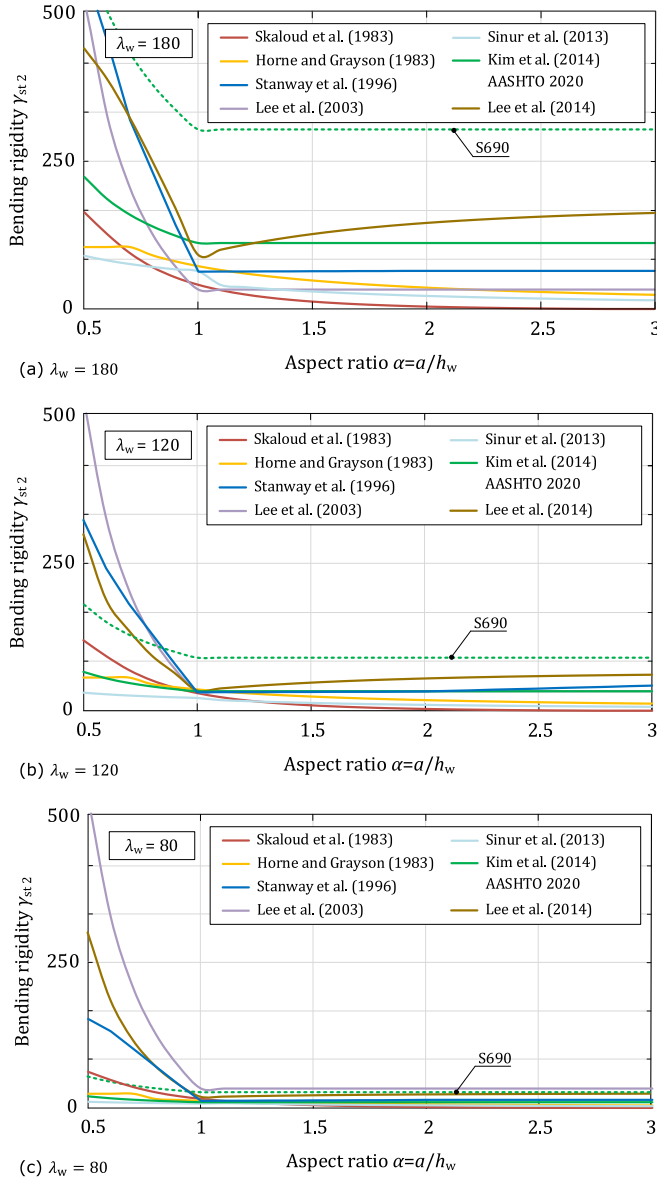


Fig. 7. Minimum ultimate bending rigidity of a transverse stiffener vs. aspect ratio according to each researcher and relevant design standards for: (a) $\lambda_w = 180$, (b) $\lambda_w = 120$, (c) $\lambda_w = 80$.

- **Local buckling:** arises when the stiffener remains stationary, dividing the web into smaller sub-panels and improving buckling performance.

For long simply supported plates under the work of [60] provides an “exact” solution for the elastic critical stress of long web panels with a single centric longitudinal stiffener. Höglund [20] proposed an approximation of these “exact” solutions. On the other hand, for shorter plates, where transverse rigid stiffeners are closely spaced, Höglund [20] and Beg [60], respectively, independently derived expressions by fitting curves to numerical results previously obtained in [46]. These expressions in Eqs. (27) and (28) form the basis of design equations in EN 1993-1-5:2024 [11] for calculating the elastic critical shear stress of longitudinally stiffened steel web panels.

$$k_\tau = 5.34 + \frac{4}{\alpha^2} + \frac{3.45 \gamma_{sl}^{3/4}}{\alpha^2} \quad (27)$$

$$k_\tau = 4.1 + \frac{6.3 + 0.05 \gamma_{sl}}{\alpha^2} + 1.44 \sqrt[3]{\gamma_{sl}} \quad (28)$$

After the acknowledgement of the shear post-buckling resistance of plate girders, in 1967, Cooper [92] proposed a model to estimate the ultimate shear resistance of longitudinally stiffened webs that assumed that each subpanel develops its own post-buckling tension field. Later in 1971, Chern and Ostapenko [93] improved Cooper’s design model to include the post-buckling frame action developed between the flanges and the longitudinal stiffener. In 1974, Rockey et al. [94] reported such behaviour in experimental testing and proposed a design method [95,96]. On the other hand, in 1975, Porter et al. [18] proposed a shear buckling resistance model in which a single tension field develops, regardless of whether a longitudinal stiffener is present.

3.1.2. Elastic critical buckling

Höglund [20] and Beg [60] expressions were modified and adopted as design rules in EN 1993-1-5:2024 [11] (Eqs. (29) and (30)), for calculating the elastic critical shear stress of longitudinally stiffened web panels. They are derived by reducing the stiffener’s second moment of area in the above expressions by 1/3 (thus, β_{sl} correction factor taken as 1). This adjustment, justified by [20,60], accounts for the lower post-buckling resistance of stiffened plates, ensuring compatibility with the shear resistance reduction curve used for unstiffened plates. This reasoning primarily applies to girders with heavy flanges, where shear resistance is often overestimated [97]. For closed cross-section stiffeners, however, the 1/3 reduction is unnecessary [1,97,98,99,100,101], as the formulation does not account for the beneficial effect of torsional rigidity, requiring a correction coefficient of $\beta_{sl}=3$.

Thus, the global elastic shear buckling coefficient for simply supported rectangular webs, with 1 or 2 longitudinal stiffeners and $a/b \geq 3$, and webs with 3 or more longitudinal stiffeners and any a/b , is given by Eq. (29). On the other hand, for simply supported rectangular webs with 1 or 2 longitudinal stiffeners and for the case of $a/b < 3$, the global elastic shear buckling coefficient is given by Eq. (30). The distance b is the total width of the stiffened panel, defined by Eq. (7).

$$k_\tau = \begin{cases} 4 + \frac{5.34}{\alpha^2} + \max \left\{ \frac{9}{\alpha^2} \sqrt[4]{\left(\frac{\beta_{sl} I_{sl}}{h_w t_w^3} \right)^3}; 2.1 \sqrt[3]{\frac{\beta_{sl} I_{sl}}{h_w t_w^3}} \right\} & \text{for } \alpha \leq 1 \\ 5.34 + \frac{4}{\alpha^2} + \max \left\{ \frac{9}{\alpha^2} \sqrt[4]{\left(\frac{\beta_{sl} I_{sl}}{h_w t_w^3} \right)^3}; 2.1 \sqrt[3]{\frac{\beta_{sl} I_{sl}}{h_w t_w^3}} \right\} & \text{for } \alpha > 1 \end{cases} \quad (29)$$

$$k_\tau = 4.10 + \frac{6.30 + 0.18 \frac{\beta_{sl} I_{sl}}{b t_w^3}}{(a/h_w)^2} + 2.20 \sqrt[3]{\frac{\beta_{sl} I_{sl}}{b t_w^3}} \quad (30)$$

3.1.3. Ultimate resistance

According to both European (EN 1993-1-5:2024 [11]) and American standards (AASHTO:2024 [4]), when calculating the ultimate shear buckling resistance of longitudinally stiffened plate girders, the longitudinal stiffeners are not explicitly taken into account in the shear resistance calculation; instead, their effect is indirectly considered through the increased elastic shear buckling stress of the web panel, consequently decreasing the web panel’s normalized slenderness. The overall method is referenced in Section 2.1.3.

3.1.4. Comparison between design models

Fig. 8 shows the comparison of the elastic shear buckling coefficient for transversely stiffened webs with and without one open longitudinal stiffener with $\gamma_{sl} = 100$ located at $h_{sl,1} = 0.5 h_w$ using the EN1993-1-5:2024 expressions [11]. Fig. 9 shows a graphical comparison of the shear buckling resistance of stiffened intermediate web panels with one open longitudinal stiffener with $\gamma_{sl} = 100$ located at $h_{sl,1} = 0.5 h_w$, calculated according to the main design methods for $\alpha = 1$ and 3 ($h_w =$

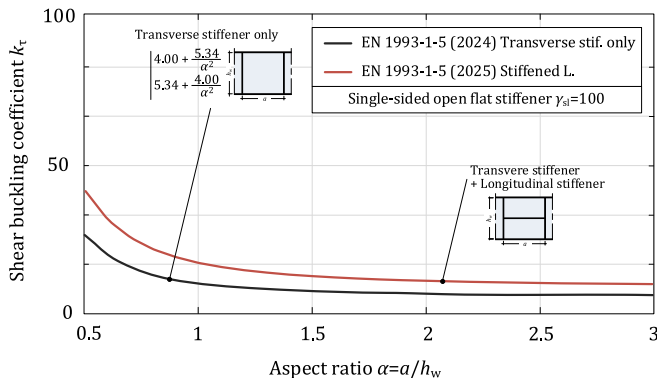


Fig. 8. Shear buckling coefficient of a longitudinally stiffened web vs. aspect ratio according to each researcher and relevant design standards.

1000 mm; $A_f/A_w = 1.0$; $b_f = 20 t_f$ and $f_{y,w} = f_{y,f} = 355$ MPa). This figure illustrates how the normalized slenderness of the web panel decreases when a longitudinal stiffener is introduced.

3.2. Design of longitudinal stiffeners

In I-girders with highly slender webs, longitudinal stiffeners are essential, as they enhance resistance to direct stresses, shear, patch loading, and improve overall load-bearing capacity. Longitudinal stiffeners help control out-of-plane deflections by increasing the web's second moment of area along their alignment. By distributing loads more uniformly, they help mitigate local buckling and increase the girder's buckling resistance [102]. Numerous studies have examined how factors such as girder depth, stiffener position and number, and combined internal forces (M–V or M–F) affect performance [100,103,104,105,106,107,108]. While these studies confirm that stiffeners improve ultimate resistance, global bending often governs failure, diminishing web resistance under shear and patch loading. In such cases, the location and quantity of stiffeners significantly influence overall buckling resistance and capacity.

Their placement generally follows the expected deformation pattern,

which is influenced by the buckling modes. These modes differ markedly from bending, shear, or patch loading buckling, making it difficult to define an optimal stiffener location under general stress conditions. Buckling behaviour also depends on the web's boundary conditions, which are influenced by flange stiffness [30,31,32]. For shear, the ideal stiffener location is at mid-depth ($h_w/2$), though a position at $h_w/5$ from the compression flange is commonly recommended in design specifications [4] and literature [109,110,111,112].

Beyond placement, a key consideration is the stiffener's minimum rigidity, which must be sufficient to control deformation and effectively subdivide the web into smaller subpanels. A longitudinal stiffener requires adequate second moment of area and cross-sectional area to establish a stable nodal line in a buckled web. If positioned in the compression zone, it must also resist axial loads. Therefore, both geometric properties are crucial in defining its dimensions, and are typically expressed using the same non-dimensional design parameters as those applied to transverse stiffeners in Section 2.

3.2.1. Research historical background

Several studies have investigated the influence of various parameters on the performance of longitudinally stiffened steel I-girders. Recent investigations have, however, focused on the optimal position of the longitudinal stiffener, especially for bend-buckling [112,113,114], rather than its required bending rigidity.

However, for thin plates under combined direct and shear stresses, Timoshenko [15] described the web's elastic behaviour, highlighting the advantages of plates stiffened by horizontal ribs. Using the energy method, he recommended the optimum relation between the cross-sectional dimension of the stiffeners and the critical values of stress in a plate. Moreover, Seide and Stein in 1949 [115] present charts and tables for calculating the critical stress of simply supported plates and having one or more longitudinal stiffeners, assuming they have zero torsional rigidity. Later in 1952, Bleich [44] investigated the stability of longitudinally stiffened web plates of girders and proposed not only a minimum stiffener bending rigidity (to reach the web plate elastic buckling load) but also assessed its optimal position [102]. Note that, the longitudinal stiffener should be positioned at the web mid-height for shear stresses, making both subpanels buckle simultaneously [102]. The

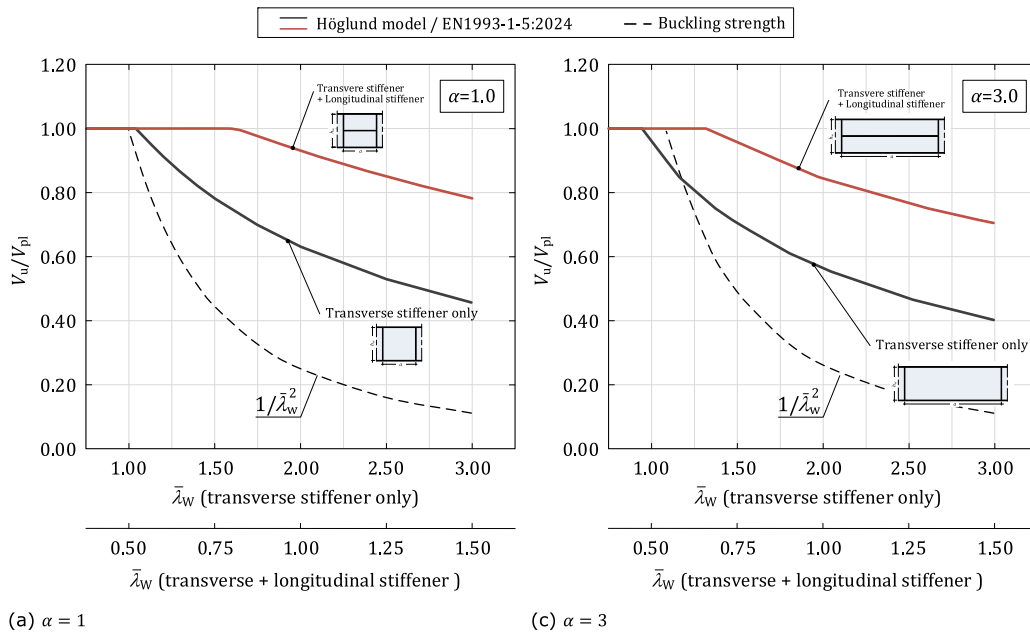


Fig. 9. Evolution of the relative shear buckling resistance of longitudinally stiffened webs V_u/V_{pl} vs. the relative web slenderness for aspect ratios: (a) $\alpha = 1$, (b) $\alpha = 3$.

plate-buckling coefficient for a square panel is $k = 9.34$, and the corresponding subpanel $k = 6.34$, and the web slenderness of the square panel is twice that of the subpanel. Consequently, the subpanel elastic shear buckling stress is 2.7 times larger than that for the square panel. Kusuda (1959) [116] investigated the influence of the torsional rigidity on longitudinal stiffeners requirements, reporting that large torsional rigidity provides partial or full restraint of the subpanel's edges, by this means increasing their critical stresses.

Recent numerical investigations have focused on the compressive resistance of stiffened plates with closed-section longitudinal stiffeners subjected to direct stresses [117]. The use of a rigid closed-section longitudinal stiffener has proven to be a highly effective solution for the design of very slender steel webs in high-strength steel, mainly to increase the ultimate shear resistance.

3.2.2. Elastic critical buckling

When the stiffeners have negligible torsional rigidity (e.g., for open stiffeners composed of a single plate, such as rectangular sections), the critical stress of the stiffened web-plate is bounded between the critical stress of an unstiffened plate of the same dimensions and the lowest critical stress of the resulting sub-plates under the resulting stress ratio. For intermediate cases, the critical stress corresponds to the critical stress of the plate-stiffener system (stiffener plus an associated width). For the design process, it is crucial to determine the stiffener's rigidity at which the elastic critical stress of the stiffened plate equals the lowest elastic critical stress of the two resulting sub-plates. Beyond this point, the critical stress does not increase, and the stiffeners become nodal lines in the plate's buckling mode. This value also depends on the aspect ratio. Generally, the smaller the aspect ratio, the smaller the required γ_{sl} value. Several methods can address this issue, ranging from solving differential equations to comprehensive numerical studies.

For the case of simply supported plates under uniform shear, it is well accepted that the stiffener placed at the central line maximises the elastic critical stress. Bleich (1952) [44] proposed Eq. (31) to compute the minimum rigidity of the stiffener at which it becomes a nodal line.

$$\gamma_{sl1} = 5.4 \alpha^2 (2\alpha + 2.5\alpha^2 - \alpha^3 - 1) \quad (31)$$

In cases where the longitudinal stiffener is placed in a different position other than the mid-height of the web due to the presence of bending (typically at $h_w/5$ from the compressed flange) the optimum rigidity is no longer given by Eq. (31). Instead, DIN 4114 (1953) [118] gives Eq. (32) when the web is under pure shear and the longitudinal stiffener is placed a $h_w/4$ from the compressed flange.

$$\gamma_{sl1} = 7.2 \alpha^2 (1 - 3.3\alpha + 3.9\alpha^2 - 1.1\alpha^3) \quad \text{for } 0.5 \leq \alpha \leq 2 \quad (32)$$

Klöppel & Scheer [46] and Klöppel & Möller [119] proposed a set of charts from which Hirt et al. [120] derived Eqs. (33) to (36) when the web is under pure shear and the longitudinal stiffener is placed a $h_w/2$, $h_w/3$, $h_w/4$ and $h_w/5$ from the compressed flange, respectively.

$$\text{For } h_w/2 \quad \gamma_{sl1} = \begin{cases} 210(\alpha - 0.4)^4 + 7.5 & \text{for } 0.7 \leq \alpha \leq 1.1 \\ 18(4.1 - \alpha)(\alpha - 2.1)^3 + 108(\alpha - 2.1) + 220 & \text{for } 1.1 \leq \alpha \leq 3.5 \end{cases} \quad (33)$$

$$\text{For } h_w/3 \quad \gamma_{sl1} = \begin{cases} (10 - 2.38\alpha)\alpha^4 & \text{for } 0.7 \leq \alpha \leq 1.6 \\ 90.6 - [35/(\alpha - 0.9)] & \text{for } 1.6 \leq \alpha \leq 3 \end{cases} \quad (34)$$

$$\text{For } h_w/4 \quad \gamma_{sl1} = \begin{cases} 4.16\alpha^4 + 5(\alpha - 1)(\alpha - 0.7) & \text{for } 0.5 \leq \alpha \leq 1.5 \\ 40 - [5.1/(\alpha - 1.2)] & \text{for } 1.5 \leq \alpha \leq 4 \end{cases} \quad (35)$$

$$\text{For } h_w/5 \quad \gamma_{sl1} = \begin{cases} 19.5(\alpha - 0.5)^3 + 0.5 & \text{for } 0.5 \leq \alpha \leq 1.3 \\ 35 - 17(2.5 - \alpha)^2 & \text{for } 1.3 \leq \alpha \leq 2.5 \end{cases} \quad (36)$$

Table 2 provides an overview of the main works related to longitudinal stiffeners, highlighting the type of loading, methodology used, and whether each work includes a design proposal for longitudinal stiffeners.

While EN 1993-1-5:2024 [11] does not specify any explicit design method for the minimum second moment of area of the longitudinal stiffener, according to AASHTO:2024 [4] the longitudinal stiffener is required to have the minimum second moment of inertia to maintain a straight line for web buckling, given in Eq. (37), recommending its position at $2 \times h_w/5$ from the compression flange in the elastic range (for composite and non-composite sections) based on Cooper's 1967 research [92].

$$I_{sl1} \geq h_w t_w^3 (2.4\alpha^2 - 0.13)\beta \Leftrightarrow \gamma_{sl1} \geq 10.92b h_w (2.4\alpha^2 - 0.13)\beta \quad (37)$$

Where β defined by Hanshin [121] is the curvature correction factor for the longitudinal stiffener rigidity given by $\beta = \frac{Z}{6} + 1$ if the longitudinal stiffener is on the side of the web away from the centre of curvature, and by $\beta = \frac{Z}{12} + 1$ if the longitudinal stiffener is on the side of the web toward the centre of curvature (where the curvature perimeter $Z = \frac{0.95 \alpha^2}{R t_w} \leq 12$ and R is the minimum girder radius in the panel).

Table 2

Overview of the investigations on longitudinal stiffeners reported in the literature.

Author	Year	Design	Load	Experimental		Numerical	Design proposal	
					Tests			Variable
Massonnet [66]	1954	U	B; S	Yes	6	No	Yes	$\gamma_{sl,2}$
Rockey [122]	1957	E; U	S	Yes	140	No	Yes	$\gamma_{sl,1}$
Klöppel and Scheer [46]	1960	E	B; S; B + S	No	-	Yes	Yes	$\gamma_{sl,1}$
Klöppel and Möller [119]	1960	E	B; S; B + S	No	-	Yes	Yes	$\gamma_{sl,1}$
Fielding and Cooper [123]	1965	E; U	S	Yes	4	No	No	-
Cooper [124]	1965	U	B; S	No	-	No	No	-
Cooper [92]	1967	U	B; S	No	-		Yes	$V_{Rd}; M_{y,Rd}$
Chern [125]	1969	U	B; S; B + S	No	-	No	Yes	$V_{Rd}; M_{y,Rd}$
Schueller and Ostapenko [126]	1970	U	B; S	Yes	4	No	No	-
Evans [127]	1986	U	B; S	Yes	2		No	-
Sinur and Beg [100]	2013	E; U	B + S	Yes	4	Yes	No	-
Kövesdi et al. [106]	2014	E; U	B + S + P	No	-	Yes	No	-
Biscaya et al. [128]	2021	E; U	B + S + C	Yes	3	No	No	-
Filho et al. [129]	2023	E	B; S	No	-	Yes	No	-

Note: E stands for elastic web buckling load design, U for ultimate web load design, S for girder shear load, B for girder overall bending, P for patch load, C for girder compression load

3.2.3. Ultimate resistance

In the assessment of longitudinally stiffened plates, EN 1993-1-5:2024 [11] states that those with weak longitudinal stiffeners should be treated as unstiffened plates concerning their resistance to direct and shear stresses. A longitudinal stiffener is classified as weak if its relative bending rigidity is less than $\gamma_{sl}=25$.

According to AASHTO:2024 [4], the longitudinal stiffener is required to have sufficient resistance to buckling of the web and to transmit the stresses in the stiffener and a portion of the web as an equivalent column [89]. Thus, the flexural stress in the longitudinal stiffeners, σ_{sl} , due to factored loads at the strength limit state, and when checking constructability, should satisfy Eq. (38) limited to the yielding strength, where the ϕ_f is the resistance factor for flexure and R_h is the hybrid factor. Eq. (38) serves as a limit on the validity of the radius of gyration of the longitudinal stiffener given in Eq. (39), where $f_{y,f,comp}$ is the yield strength of the compression flange (in ksi).

$$\sigma_{sl} \leq \phi_f R_h f_{y,sl} \quad (38)$$

$$i \geq \frac{0.16 a \sqrt{f_{y,sl}/E}}{\sqrt{1 - 0.6 \frac{f_{y,f,comp}}{R_h f_{y,sl}}}} \quad (39)$$

3.2.4. Effective stiffener cross section

When calculating the longitudinal stiffener mechanical properties, namely its bending rigidity, a portion of the web should be included as part of the stiffener's cross-section. This is in line with the value assumed in EN 1993-1-5:2024 [11], where the second moment of area of the longitudinal stiffener is computed considering the effective cross-section, including a participating width of the web of $b_{w,eff} = 10 t_w$ on each side of the longitudinal stiffener, as in Fig. 5. Furthermore, AASHTO:2024 [4] specify that the second moment of area and radius of gyration of the longitudinal stiffener should also consider the effective cross-section, taking a $b_{w,eff} = 9 t_w$ on each side of the longitudinal stiffener, and should be calculated about the neutral axis of the combined section [89].

3.2.5. Comparison between design models

Fig. 10 compares the key proposals for the bending rigidity of longitudinal stiffeners, γ_{sl1} , placed at $h_w/2$ designed for elastic web behaviour when subjected to shear loads. No rules for the ultimate bending rigidity of longitudinal stiffeners of plate girder subjected to bending and/or shear loads were found.

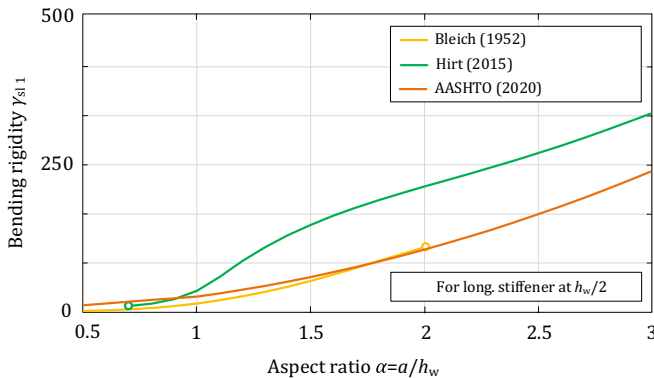


Fig. 10. Minimum elastic bending rigidity γ_{sl1} of a longitudinal stiffener vs. aspect ratio according to each researcher and relevant design standards.

4. Diagonally stiffened webs

Diagonally stiffened webs are not commonly adopted in design practice. Their use may raise construction-related issues, particularly concerning the welding of stiffeners at the corners, which must be carefully addressed. Nevertheless, several authors have proposed this configuration as an alternative to the conventional transverse and longitudinally stiffened webs. The aim is therefore to review the research on this alternative form of web stiffening and its potential to enhance the ultimate shear resistance.

4.1. Shear buckling resistance of diagonally stiffened webs

This section provides a focused review of the behaviour and design methodologies of diagonally stiffened web panels, highlighting the latest research trends and the ranges of key parameters explored thus far. It begins with an overview of historical references, followed by a detailed discussion of existing calculation models. These models address two primary aspects: (i) predicting the elastic critical buckling load and (ii) predicting the ultimate shear resistance of diagonally stiffened I-girders. Finally, where possible, the available design models are compared and their limitations discussed.

4.1.1. Research historical background

Research on the buckling behaviour of diagonally stiffened plates began nearly a century ago, progressing alongside studies on the shear behaviour of unstiffened slender panels. While this paper focuses on diagonally stiffened web panels, understanding the fundamental shear mechanisms of unstiffened panels provides essential context for interpreting the behaviour of stiffened panels. A brief overview of these mechanisms is presented below.

The understanding of fundamental shear mechanisms in web panels, including load paths and stress distributions established in the 1930s, laid the foundation for the use of diagonal stiffeners. These stiffeners were introduced with two primary objectives: to increase the critical shear buckling load and to enhance ultimate shear capacity in the post-buckling range, akin to the role of diagonals in a Pratt truss. Early studies [130,131] demonstrated that placing a diagonal stiffener along the compression diagonal significantly improves both buckling resistance and ultimate shear capacity.

After several decades of limited progress, research on diagonal stiffeners saw a resurgence in the 1970s and 1980s, driven by advancements in tension field theories that shifted the focus from critical shear load (V_{cr}) to the ultimate shear capacity (V_u). Numerous experimental studies from this period [132,133,134,135,136,137,138] investigated the influence of diagonal stiffeners on pre- and post-buckling behaviour. Table 3 summarizes key experimental studies that have contributed to improving girder shear performance. It highlights authors who proposed expressions for predicting either the critical buckling load (V_{cr}) or the ultimate resistance (V_u) of diagonally stiffened girders.

Yonezawa et al. [134] developed expressions for the shear buckling coefficient (k_τ) under various diagonal orientations and boundary conditions, including simply supported and clamped edges. Complementary experiments by Jetteur [137] revealed that diagonally braced I-girders behave similarly to unbraced beams, with tension field formation in the web and plastic hinge development in the flanges at failure. These studies significantly advanced the understanding of slender webs with diagonal stiffeners and laid the groundwork for modern design methodologies, which are further detailed in Sections 4.2 and 4.3.

Most recently, in 2024, Martins et al. [139] conducted experimental tests on steel I-girders with diagonal stiffeners, exploring different

Table 3

Overview of the investigations on diagonal stiffeners reported in the literature.

Author	Year	Design	Load	Direction	Experimental		Numerical	Design proposal		
						Tests			Variable	Rigidity
Miakami et al. [132]	1971	E	S; B	Open C, T	Yes	2	No	No	-	-
Yonezawa et al. [134]	1978	E; U	S	Open C, T	Yes	2	No	Yes	k_τ	-
Cescotto et al. [135]	1980	U	S	Open X	Yes	6	No	No	-	-
Jetteur [136,137]	1982	U	S	Open C	Yes	8	No	Yes	V_u	-
Guarnieri [138]	1985	U	S; B	Open C	Yes	98	No	Yes	V_u	-
Alavi & Nateghi [145]	2013	U	S	Open X	Yes	3	Yes	Yes	V_u	-
Azmi et al. [146]	2017	U	S; B	Open C	Yes	10	Yes	No	-	-
Yuan et al. [140]	2019	E; U	S	Open C	Yes	2	Yes	Yes	k_τ ; V_u	Open (γ_{sd})
Hayatdavoodi et al. [147]	2020	U	S	Open C, T, X	No	-	Yes	Yes	V_u	-
Martins et al. [139,141,142]	2022	E	S	Open/Closed C	Yes	3	Yes	Yes	k_τ	Open (γ_{sd} ; θ_{sd})
Yang et al. [143,144]	2022	E	S; B	Closed C	No	-	Yes	Yes	k_τ ; k	Closed (γ_{sd} ; θ_{sd})

Note: E stands for elastic web buckling load design, U for ultimate web load design, S for girder shear load, B for girder overall bending, C for stiffener along the compression diagonal, T along the tension diagonal, and X along both

stiffener properties and joining techniques (welding and bonded-bolting). The results confirmed that diagonal stiffeners enhance shear capacity, though bonded-bolting stiffeners exhibited reduced ductility. A qualitative comparison of three existing design models (detailed in Section 4.3) revealed substantial scatter in predicting ultimate shear resistance, emphasising the need for improved methodologies.

In recent years, the accessibility of advanced FE tools has reignited interest in this topic, leading to a surge in numerical and experimental studies (see Table 3), which revisited key issues, including: (i) shear buckling stress and coefficients; (ii) ultimate shear resistance and the validity of assumptions regarding stiffener plastification; (iii) shear failure mechanisms and post-buckling stress distribution; (iv) effects of stiffener flexural and torsional rigidity.

Significant progress has also been made in refining formulations for the shear buckling coefficient, k_τ . Yuan et al. [140] introduced bending rigidity considerations, while Martins and Cardoso [141,142] extended these models to include torsional rigidity. Recently, Yang et al. [143] investigated the elastic buckling behaviour of diagonally stiffened steel plate shear walls under combined shear and non-uniform compression, focusing on closed C-shaped stiffeners. The authors proposed interaction curves for panels under combined stresses, and in a follow-up study [144], analysed the effects of axial forces alongside non-uniform bending and shear.

4.1.2. Elastic critical buckling

In 1978, Yonezawa et al. [134] conducted in-depth investigations on diagonally stiffened plate girders, with different diagonal directions and boundary conditions (i.e., with simply supported and clamped all four edges). For simply supported panels, the authors proposed Eq. (40) for the shear buckling coefficient k_τ for both diagonal directions, expressed as a function of the aspect ratio only.

$$k_\tau = \begin{cases} 11.9 + \frac{10.1}{\alpha} + \frac{10.9}{\alpha^2} & \text{for stiffener in compression diagonal} \\ 17.2 - \frac{22.5}{\alpha} + \frac{16.7}{\alpha^2} & \text{for stiffener in tension diagonal} \end{cases} \quad (40)$$

In 2019, Yuan et al. [140] proposed a set of expressions, derived from regression analysis of numerical results, specifically for stainless steel web panels stiffened on both sides along the compression diagonal, as

given in Eq. (41). A key advancement in these expressions is the inclusion of the stiffener's relative flexural rigidity, represented by its second moment of area ($I_{sd,y}$), calculated with respect to the mid-plane of the web panel. This calculation accounts for adjacent portions of the web plate, equal to $b_{w,eff} = 11 \varepsilon t_w$ as in Fig. 5. The authors concluded that the buckling coefficient increases with the flexural rigidity of diagonal stiffeners, with this effect being particularly pronounced for smaller web aspect ratios and diagonal angles closer to 45° .

$$k_\tau = \begin{cases} \left(\frac{24.2}{\alpha} - 2\right)^3 \sqrt{\frac{I_{sd,y}}{h_w t_w^3}} + \frac{6.57}{\alpha^2} + 4.92 & \text{for } \frac{I_{sd,y}}{h_w t_w^3} < 1 \text{ and } \alpha < 1 \\ \left(\frac{24.2}{\alpha} - 2\right)^3 \sqrt{\frac{I_{sd,y}}{h_w t_w^3}} + \frac{4.92}{\alpha^2} + 6.57 & \text{for } \frac{I_{sd,y}}{h_w t_w^3} < 1 \text{ and } \alpha \geq 1 \\ \left(\frac{9}{\alpha} - 2\right)^3 \sqrt{\frac{I_{sd,y}}{h_w t_w^3}} + \frac{6.57}{\alpha^2} + \frac{15.2}{\alpha} + 4.92 & \text{for } \frac{I_{sd,y}}{h_w t_w^3} \geq 1 \text{ and } \alpha < 1 \\ \left(\frac{9}{\alpha} - 2\right)^3 \sqrt{\frac{I_{sd,y}}{h_w t_w^3}} + \frac{4.92}{\alpha^2} + \frac{15.2}{\alpha} + 6.57 & \text{for } \frac{I_{sd,y}}{h_w t_w^3} \geq 1 \text{ and } \alpha \geq 1 \end{cases} \quad (41)$$

Building upon this contribution, Martins and Cardoso [142] went a step further and developed a similar analytical framework that also considered stiffeners with closed cross-sections, thereby accounting for the torsional rigidity of the stiffeners. In their study, the authors additionally overcame another limitation of the previous work by extending the formulation to panels surrounded by upper and lower flanges and vertical stiffeners with semi-rigid rotational restraints on all four edges, rather than assuming simply supported boundary conditions typically adopted in standard design methods. Based on extensive numerical analyses, they proposed separate expression for open (Eq. (42)) and closed (Eq. (43)) cross-section diagonal stiffeners placed along the compression diagonal. In these expressions, $\gamma_{sd,y}$, $\gamma_{sd,z}$, $\theta_{sd,x}$ are relative flexural rigidity around the strong axis, relative flexural rigidity around the weak axis, and relative torsional rigidity of the stiffener, obtained using Eqs. (4) and (5) and taking $b = h_w$ and $J_s = I_T$. It is worth noting that in their expression, the stiffener's second moment of area, $I_{sd,y}$, is calculated with respect to the web's mid-line; however, unlike Yuan's model and the practice adopted in EN1993-1-5:2024 [11], the contribution of the adjacent web plates is disregarded; while $I_{sd,z}$ is determined with

reference to the centre of mass of the stiffener alone. Similarly, I_T denotes the torsional constant of the stiffener, also computed with reference to the centre of mass of the stiffener alone.

According to [18], θ_D represents the inclination of the panel diagonal, and φ denotes the inclination of the tension field, which can be determined iteratively, until the maximum value of shear resistance is obtained. The term $M_{f,pl}$ refers to the plastic bending moment formed in

$$k_\tau = \begin{cases} 5.23 \frac{\gamma_{sd,y}^{0.404} \gamma_{sd,z}^{0.038}}{\alpha^{1.371}} + \frac{11.50}{\sqrt[3]{\gamma_{sd,y}}} + \frac{5.34}{\alpha^2} + 4.00 & \text{for } \gamma_{sd,y} \leq 12(1 - \nu^2) \text{ and } 0.5 < \alpha \leq 1 \\ 6.15 \frac{\gamma_{sd,y}^{0.428} \gamma_{sd,z}^{0.020}}{\alpha^{1.434}} + \frac{4.07}{\sqrt[3]{\gamma_{sd,y}}} + \frac{4.00}{\alpha^2} + 5.34 & \text{for } \gamma_{sd,y} \leq 12(1 - \nu^2) \text{ and } 1 < \alpha \leq 2 \\ 10.16 \frac{\gamma_{sd,y}^{0.026} \gamma_{sd,z}^{0.076}}{\alpha^{1.788}} - \frac{17.96}{\sqrt[3]{\gamma_{sd,y}}} + \frac{3.35}{\alpha^2} + \frac{5.67}{\alpha} + 18.35 & \text{for } \gamma_{sd,y} > 12(1 - \nu^2) \text{ and } 0.5 < \alpha \leq 1 \\ 6.92 \frac{\gamma_{sd,y}^{-0.036} \gamma_{sd,z}^{0.375}}{\alpha^{1.632}} - \frac{14.29}{\sqrt[3]{\gamma_{sd,y}}} + \frac{11.45}{\alpha^2} + \frac{9.96}{\alpha} + 12.03 & \text{for } \gamma_{sd,y} > 12(1 - \nu^2) \text{ and } 1 < \alpha \leq 2 \end{cases} \quad (42)$$

$$k_\tau = \begin{cases} 5.95 \frac{\gamma_{sd,y}^{0.344} \gamma_{sd,z}^{0.086}}{\alpha^{1.557}} + \frac{1.46}{\sqrt[3]{\theta_{sd,x}}} + \frac{5.03}{\sqrt[3]{\gamma_{sd,y}}} + \frac{5.34}{\alpha^2} + 4.00 & \text{for } \gamma_{sd,y} \leq 12(1 - \nu^2) \text{ and } 0.5 < \alpha \leq 1 \\ 6.81 \frac{\gamma_{sd,y}^{0.395} \gamma_{sd,z}^{0.011}}{\alpha^{1.083}} - \frac{1.25}{\sqrt[3]{\theta_{sd,x}}} + \frac{3.85}{\sqrt[3]{\gamma_{sd,y}}} + \frac{4.00}{\alpha^2} + 5.34 & \text{for } \gamma_{sd,y} \leq 12(1 - \nu^2) \text{ and } 1 < \alpha \leq 2 \\ 2.19 \frac{\gamma_{sd,y}^{-0.211} \gamma_{sd,z}^{0.641}}{\alpha^{1.942}} - \frac{64.40}{\sqrt[3]{\theta_{sd,x}}} + \frac{56.50}{\sqrt[3]{\gamma_{sd,z}}} - \frac{17.01}{\sqrt[3]{\gamma_{sd,y}}} + \frac{13.62}{\alpha^2} + 24.49 & \text{for } \gamma_{sd,y} > 12(1 - \nu^2) \text{ and } 0.5 < \alpha \leq 1 \\ 2.95 \frac{\gamma_{sd,y}^{-0.170} \gamma_{sd,z}^{0.563}}{\alpha^{1.578}} - \frac{29.04}{\sqrt[3]{\theta_{sd,x}}} + \frac{25.62}{\sqrt[3]{\gamma_{sd,z}}} - \frac{10.70}{\sqrt[3]{\gamma_{sd,y}}} + \frac{16.48}{\alpha^2} + 17.34 & \text{for } \gamma_{sd,y} > 12(1 - \nu^2) \text{ and } 1 < \alpha \leq 2 \end{cases} \quad (43)$$

4.1.3. Ultimate resistance

As shown in Table 3, there are currently 3 design model proposals for the prediction of the ultimate resistance of a diagonally stiffened web panel – from Jetteur [136,137], Yuan et al. [140], and Hayatdavoodi et al. [147]. It is important to note that the design expressions by Guarnieri [138] and Alavi and Nateghi [145] are excluded from this analysis, as Alavi's model is identical to that of Hayatdavoodi et al., while Guarnieri's model applies only to web panels with stiffeners slightly inclined from the vertical axis, rather than fully diagonally stiffened panels.

In Jetteur's model of 1982, the effect of the diagonal stiffener is considered in both the elastic buckling load and the post-critical collapse mechanism. To quantify the elastic buckling load (V_{cr}), the authors utilized the up-to-date tables provided by Ebel in 1973 [133]. For the post-critical behaviour of the girder, the authors adopted the Porter method without significant modifications. The following assumptions were made to incorporate the diagonal stiffener's effect: (i) The contribution of the diagonal stiffener (V_d) is directly added to the remaining load-carrying components of the Porter model for an unstiffened panel (V_{cr} and V_t); (ii) The diagonal stiffener is assumed to be fully yielded, meaning that its buckling is prevented by the web panel, and that it possesses sufficient flexural inertia; (iii) The tension field and its membrane stress component (σ_t) are unaffected by the presence of the compressed diagonal stiffener. The stress distribution above and below the stiffener remains unchanged, with no interference or alteration. Considering these assumptions, by summing the components according to the Porter model (see Eq. (8)), the total shear capacity for a diagonally stiffened panel is obtained using Eq. (44) as:

$$V_u = \underbrace{h_w t_w \tau_{cr}}_{V_{cr}} + \underbrace{c t_w \sigma_t \sin^2 \varphi (\cot \varphi - \cot \theta_D)}_{V_t} + \frac{4M_{f,pl}}{c} + \underbrace{P_{pl,sd} \sin \theta_D}_{V_d} \quad (44)$$

the flanges ($W_{pl,f} f_{y,f}$ where $f_{y,f}$ denote the yield strengths of the flanges), while $P_{pl,sd}$ represents the plastic resistance of the stiffener ($A_{sd} f_{y,sd}$). Finally, the membrane stress field (σ_t) is calculated using Eq. (45), applying the von Mises yield criterion. The distance c defines the position of the plastic hinge in the flange and depends on the plastic moment of the flange $M_{f,pl}$. It can be determined using plastic hinge theory through the principle of virtual work. For equal top and bottom flanges, c can be calculated using Eq. (46).

$$\sigma_t = \sqrt{f_{y,w}^2 - \tau_{cr}^2} \left[3 - \left(\frac{3}{2} \sin 2\varphi \right)^2 \right] - \frac{3}{2} \tau_{cr} \sin 2\varphi \quad (45)$$

$$c = \frac{2}{\sin \theta} \sqrt{\frac{M_{f,pl}}{\sigma_t t_w}} \quad (46)$$

Furthermore, the design model proposed by Yuan et al. [140] integrates Höglund's rotated stress field model [19] with earlier design approaches from the 1970s and 1980s for diagonally stiffened plates. In this model, the contribution of the diagonal stiffener is consistent with the Porter model (see Eq. (8)) and the shear resistance (V_u) is calculated using Eq. (47).

$$V_u = V_{b,w} + V_{b,f} + V_d \quad (47)$$

In this model, $V_{b,w}$, $V_{b,f}$, and V_d represent the shear resistance contributions of the web (unstiffened or stiffened), the flanges, and the symmetrical double-sided diagonal stiffener, respectively. The first two components can be determined using the current design provisions of EN 1993-1-5:2024 [11] and EN 1993-1-4+A1:2015 [148], which predict the shear resistance of carbon steel and stainless-steel plate girders. For carbon steel structures, which are the focus of this paper, the web contribution $V_{b,w}$ is calculated using Eq. (12). In this expression, the

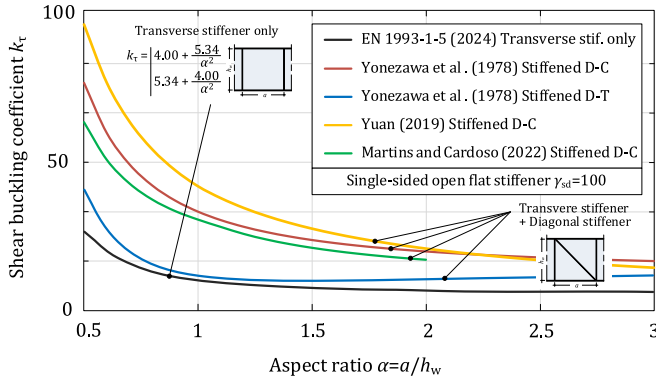


Fig. 11. Shear buckling coefficient of a diagonally stiffened web vs. aspect ratio according to each researcher and relevant design standards

partial factor is $\gamma_{M1}=1.0$, and the reduction factor (χ_w) is determined using Eq. (13) (under the assumption of a non-rigid end post). A recommended value of $\eta = 1.0$ is applied by the same authors. On the other hand, the flange contribution to shear resistance, $V_{b,f}$, is calculated using Eq. (15) where M_{Ed} and $M_{f,Rk}$ represent the acting design bending moment and the design plastic moment of resistance of the flanges, respectively, with the condition $M_{Ed} < M_{f,Rk}$. The third component, V_d , is calculated using Eq. (48) as proposed by Jensen and Antoni [131], under the assumption that the diagonal stiffener undergoes full plastification. According to the authors, this assumption aligns with the observations made in two experimental tests.

$$V_d = A_{sd} f_{y, sd} \sin \theta_D \quad (48)$$

According to the model proposed by Hayatdavoodi et al. in 2020 [147], the ultimate shear resistance of diagonally stiffened girders according to this model is calculated using Eq. (49):

$$V_u = \underbrace{h_w t_w \tau_{cr}}_{V_{cr}} + \underbrace{t_w \sigma_t \sin^2 \varphi (h_w (\cot \varphi - \cot \theta_D) + (c_c + c_t))}_{V_t} + \underbrace{A_{sd} \sin \theta_D (\sigma_{sd,c} + \sigma_{sd,t})}_{V_d} \quad (49)$$

The critical shear stress (τ_{cr}) is determined with the shear buckling coefficient (k_t) expression proposed by Yonezawa et al. [134], given by Eq. (40). The area of the diagonal stiffeners (A_{sd}) assumes identical stiffeners in both the compression and tension directions. The tensile and compressive stresses in the diagonal stiffeners are denoted as $\sigma_{sd,t}$

and $\sigma_{sd,c}$, respectively. Since the stiffener constrains the plate along its contact line, the normal stresses acting perpendicular to the tensile field can be expressed using Hooke's law as a function of the tensile membrane component: $\nu \times \sigma_t$. By superimposing the critical shear stress (τ_{cr}) with the tensile membrane stress (σ_t) – derived from Eq. (45) – and applying trigonometric transformations and Mohr's circle analysis as outlined in Timoshenko [15], $\sigma_{sd,t}$ and $\sigma_{sd,c}$ can be expressed using Eqs. (50) and (51). Here, $\sigma_{cr, sd}$ is the buckling stress of the diagonal stiffeners, calculated using Eq. (52) according to Yonezawa et al. [134]:

$$\sigma_{sd,t} = \sigma_t [1 - (1 + \nu) \sin^2 (\theta_D - \varphi)] + [(1 + \nu) \tau_{cr} \sin 2\theta_D] \leq f_{y, sd} \quad (50)$$

$$\sigma_{sd,c} = -\sigma_t [1 - (1 + \nu) \sin^2 (\theta_D + \varphi)] + [(1 + \nu) \tau_{cr} \sin 2\theta_D] \leq \sigma_{cr, sd} \quad (51)$$

$$\sigma_{cr, sd} = \begin{cases} f_{y, sd} & \text{for } \bar{\lambda}_{sd} < 0.45 \\ f_{y, sd} \left(1 - 0.53 \left(\bar{\lambda}_{sd} - 0.45 \right)^{1.36} \right) & \text{for } 0.45 \leq \bar{\lambda}_{sd} < \sqrt{2} \\ \frac{f_{y, sd}}{\bar{\lambda}_{sd}^2} & \text{for } \bar{\lambda}_{sd} \geq \sqrt{2} \end{cases} \quad (52)$$

The relative slenderness of the stiffener and the critical buckling coefficient are obtained from Eqs. (53) and (54), where l is the effective length of the diagonal stiffeners:

$$\bar{\lambda}_{sd} = \frac{b_{sd}}{t_{sd}} \sqrt{\frac{f_{y, sd}}{E} \frac{12(1 - \nu^2)}{\pi^2 k_{sd}}} \quad (53)$$

$$k_{sd} = \left(\frac{b_{sd}}{l} \right)^2 + 0.425 \quad (54)$$

To prevent local buckling of the diagonal stiffeners, the authors recommend maintaining a slenderness ratio of $b_{sd}/t_{sd} \leq 0.56 \sqrt{E/f_{y, st}}$, as adopted also by AISC [149].

4.1.4. Comparison between design models

Fig. 11 shows the comparison between the proposals for the shear buckling coefficient of an open diagonal stiffened web ($\gamma_{sd,y} = 100$ and $\gamma_{sd,z} = 0.001$) and the EN 1993-1-5:2024 expression for a web panel with transverse stiffeners only [11]. The addition of the stiffener along the compression diagonal (C) significantly increases the buckling coefficient regardless of the aspect ratio, whereas the stiffener placed in the tensile

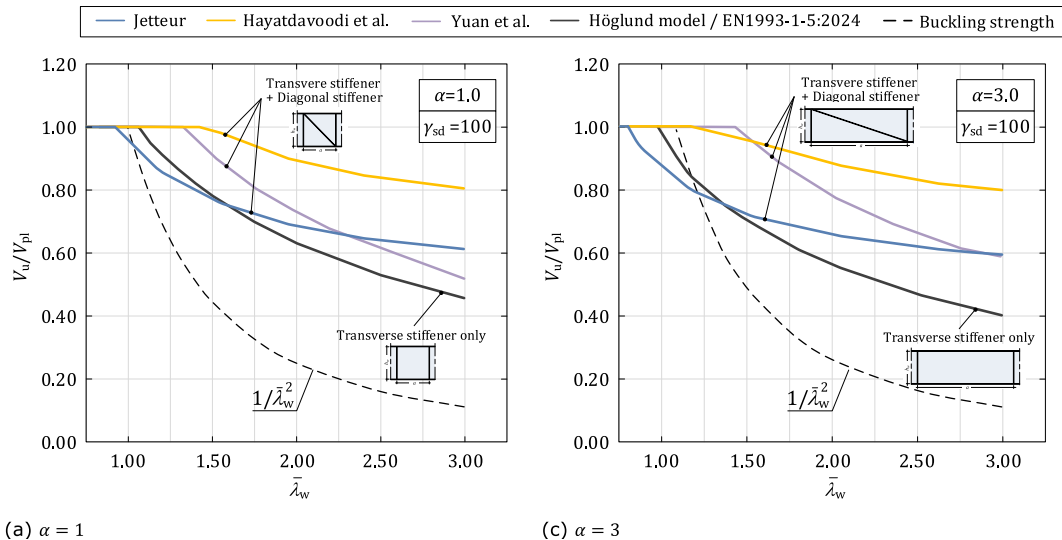


Fig. 12. Evolution of the relative shear buckling resistance of diagonally stiffened webs V_u/V_{pl} vs. the relative web slenderness for aspect ratios: (a) $\alpha = 1$, (b) $\alpha = 3$

stress direction (T) has only a marginal effect. Furthermore, the differences between the three models (i.e., Yonezawa, Yuan, Martins and Cardoso) are also non-negligible. These discrepancies arise in part from how the stiffener's second moment of area is calculated. Yonezawa's model does not consider it at all. Yuan's formulation is derived for web panels with double-sided diagonal stiffeners, incorporating the contribution of adjacent web plates, and therefore does not adequately represent cases with single-sided stiffeners. In contrast, Martins and Cardoso model excludes the adjacent plate contribution in the second moment of area and is limited to aspect ratios below 2, restricting its applicability to longer panels.

Fig. 12 shows a graphical comparison of the shear buckling resistance of stiffened intermediate web panels with one open diagonal stiffener with $\gamma_{sd,y} = 100$ and $\gamma_{sd,z} = 0.001$ located along the compressed diagonal, calculated according to the main design methods for $\alpha = 1$ and 3 ($h_w = 1000$ mm; $A_f/A_w = 1.0$; $b_f = 20$ t_f and $f_{y,w} = f_{y,f} = 355$ MPa).

4.2. Design of diagonal stiffeners

Diagonal stiffeners play a critical role in enhancing the shear resistance of I-girders, particularly by increasing the critical shear buckling load of the web panel and contributing to the load-carrying capacity in the post-buckling range [150]. By incorporating diagonal elements within the web, as illustrated in Fig. 2 (c), this technique transforms the girder into a hybrid structural system, combining the characteristics of a truss and an I-girder. Although the potential of diagonal stiffeners to improve stability and load capacity has been recognised for decades, their inclusion has yet to be adopted in current design standards. However, recent advances in computational techniques have reignited interest in this topic, leading to significant developments in the understanding and design of diagonally stiffened I-girders.

4.2.1. Research historical background

Although diagonal web stiffeners share a design philosophy with longitudinal stiffeners, there are currently no specific design guidelines design to them. Most existing studies on diagonally stiffened I-girders focus on their global contribution to shear resistance, overlooking local design aspects such as sizing, slenderness, and buckling verification. Consequently, to date, no dedicated formulations exist for the design or verification of diagonal stiffeners covering factors like bending or torsional rigidity. As a result, current models often rely on the same assumptions used for longitudinal stiffeners.

Accordingly, detailed discussions on elastic critical buckling, ultimate resistance, and comparative design models are omitted. However, selected insights from historical studies on effective stiffener cross-sections are presented in the following section.

4.2.2. Effective stiffener cross section

In the case of diagonal stiffeners, the definition of the effective stiffener cross-section varies across different studies, depending on the design model and underlying assumptions. In models of Jetteur [137] and Hayatdavoodi et al. [147] (based on Porter et al.), only the gross cross-section of the stiffener is considered, without including any portion of the web plate. Similarly, in the model developed by Martins and Cardoso [142] for evaluating the elastic critical buckling of stiffened web panels, the effective cross-section includes the gross area of the stiffener only, whereby the second moment of area of the stiffener is calculated about the mid-plane of the web. Finally, Yuan et al. [140] consider the adjacent web plate, with an effective width of $b_{w,eff} = 11 \epsilon t_w$, following the provisions of EN 1993-1-4 [148] for stainless steel members, as illustrated in Fig. 5.

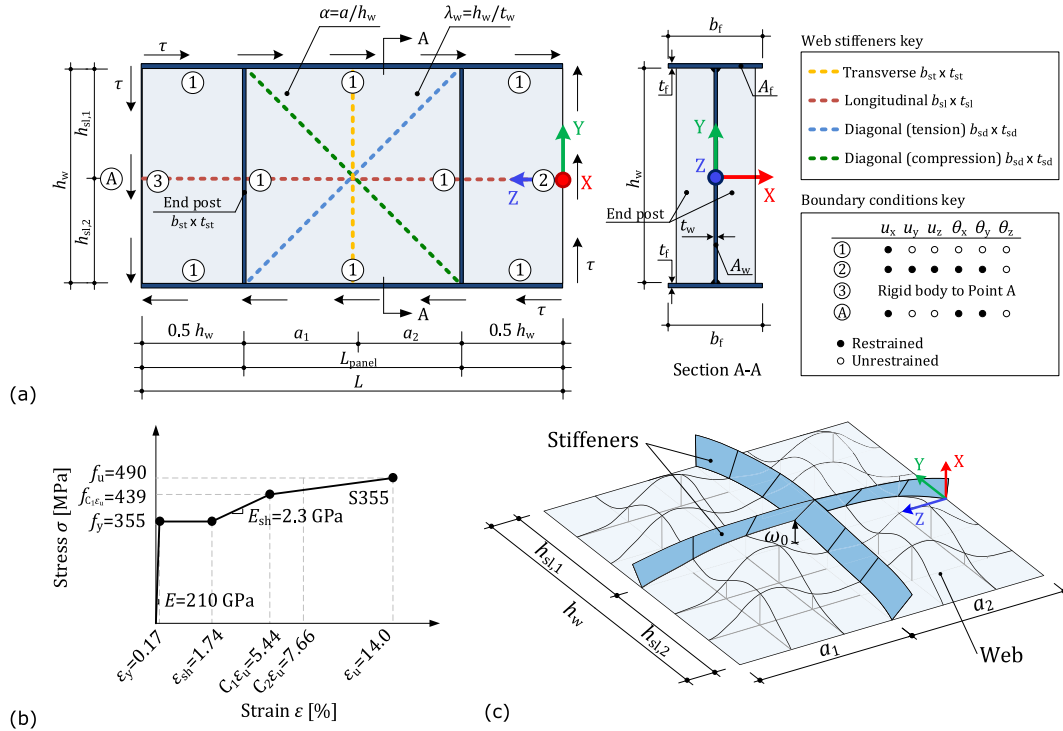


Fig. 13. FE-model: (a) Geometry and boundary conditions, (b) material model, (c) example of equivalent initial geometric imperfections.

Table 4

FE results of elastic shear buckling, ultimate load and increase in resistance.

Variable		U1	T1	L1	DT1	DC1	U3	T3	L3	DT3	DC3
$k_{t,FEM}$	[-]	0.22	0.64	0.57	0.31	0.73	0.14	0.17	0.50	0.22	0.29
$V_{cr,FEM}$	[kN]	180	524	467	254	598	115	130	410	180	238
$V_{u,FEM}$	[kN]	546	754	613	700	761	342	434	500	394	568
V_{pl}	[kN]	820	820	820	820	820	820	820	820	820	820
$V_{u,FEM}/V_{pl}$	[-]	0.67	0.90	0.75	0.85	0.93	0.42	0.53	0.61	0.48	0.69
V_u Increase	[%]	-	+35%	+12%	+28%	+39%	-	+27%	+46%	+15%	+66%

5. Comparison with finite element results

5.1. Numerical model

In this section, a finite element (FE) analysis is carried out using the ABAQUS software [154] on a symmetric three-panel I-shaped plate girder to illustrate the influence of different stiffener configurations on shear buckling resistance and out-of-plane web displacements. The numerical study includes girders with unstiffened webs as well as webs stiffened with transverse, longitudinal, or diagonal stiffeners, allowing a direct comparison of their structural response. The FE model was validated by replicating experimental tests [155], and a comprehensive comparison between the numerical and analytical results was reported in [34,89]. The main objective of the FE analysis presented hereafter is to provide an indicative comparison with the analytical methods discussed in previous sections. The following paragraphs present the details of the validated FE model adopted in this study.

The web panel height and thickness are fixed at $h_w = 1000$ mm and with $t_w = 4$ mm, and the central web panel with a width L_{panel} is limited by end post rigid flat transverse stiffeners. Flange dimensions with

$A_f/A_w = 1.0$ are adopted considering $b_f = 20 t_f$. Together with the end posts, the application of a single-sided flat transverse stiffener was investigated by means of an intermediate transverse stiffener ($\gamma_{st} = 300$ or 100 depending on α [89]; $b_{st}/t_{st} = 10$) – in yellow in Fig. 13 (a). The influence of an open longitudinal stiffener was explored ($\gamma_{sl} = 100$; $b_{sl}/t_{sl} = 10$) together with end post stiffeners – in red in Fig. 13 (a). Moreover, the advantages of stiffening the web plate with a single-sided open diagonal stiffener, together with the end post stiffeners, were assessed ($\gamma_{sd} = 100$; $b_{sd}/t_{sd} = 10$) for both compression and tension diagonal stiffening cases, respectively, in green and blue in Fig. 13 (a). Furthermore, two distinct central web panel lengths were investigated: short ($L_{panel} = h_w$) and long web panels ($L_{panel} = 3 \times h_w$), respectively, with aspect ratios $\alpha = L_{panel}/h_w$ of 1 and 3. The boundary conditions are represented in Fig. 13 (a) and were designed to replicate shear loading accurately. All parameters align with current research on plate structures [151,152].

A linear S4R shell element with four nodes, first-order interpolation, and reduced integration was used. A mesh convergence study recommended a maximum element size of 40×40 mm ($\approx h_w/25$) to balance accuracy and efficiency [89]. Shell cross-sectional behaviour was

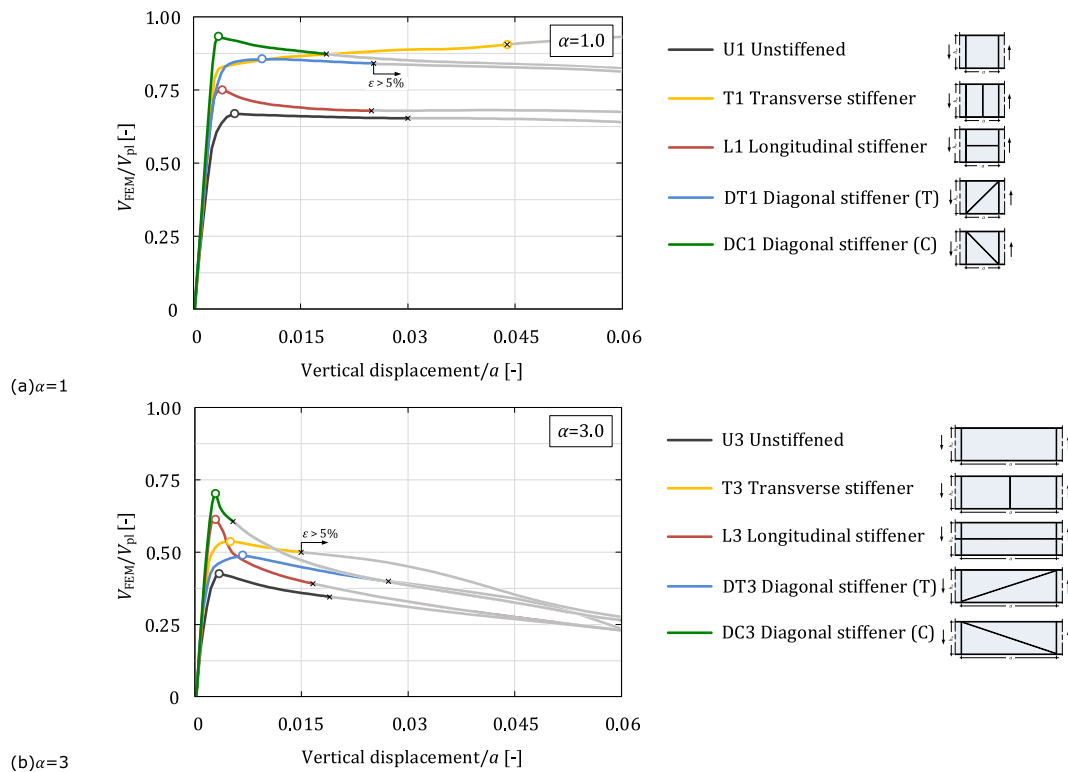


Fig. 14. FE normalised shear resistance vs. vertical displacement for: (a) $\alpha = 1$, (b) $\alpha = 3$.

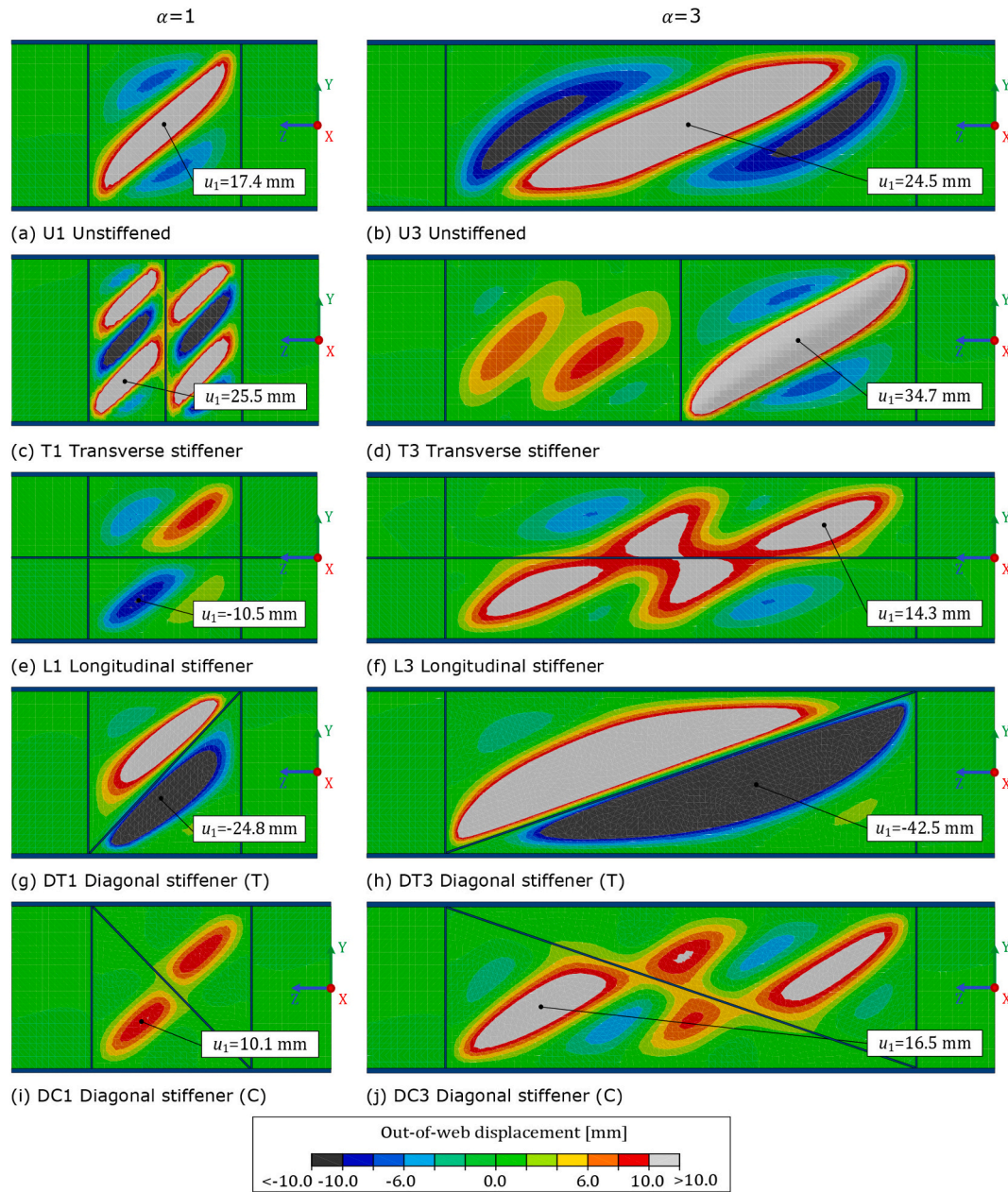


Fig. 15. FE out-of-web displacements for the maximum shear V_{FEM} for: (a) U1, (b) U3, (c) T1, (d) T3, (e) L1, (f) L3, (g) DT1, (h) DT3, (i) DC1, (j) DC3.

evaluated using Simpson's rule with five integration points. Plus, two analyses were performed on the same finite element model: 1) Linear Bifurcation Analysis (LBA) to assess local geometric imperfections and elastic buckling loads, and 2) Geometric and Material Nonlinear Analysis (GMNIA), including web and stiffener equivalent imperfections to determine girder ultimate resistance. The arc-length (Riks) method was used to solve the nonlinear system, with static edge loads applied to capture collapse and post-buckling behaviour. Moreover, as in FprEN 1993-1-14 [153], the ultimate shear resistance V_u was defined as the lesser of: 1) the peak shear load from the load-deformation curve, or 2) the shear load at which the equivalent strain reaches the limit $\epsilon_{lim}=5\%$, as recommended for plated structures [11].

The study examined the behaviour of S355 JR structural steel under static loading, assuming an identical tension-compression response together with von Mises yield criterion. An elasto-plastic material model was applied using a quad-linear strain-hardening model with a Young's modulus of 210 GPa and nominal material properties (Fig. 13(b)).

The numerical study considered equivalent geometric imperfections in line with FprEN 1993-1-14 [153], combining eigenmode-based shapes and sinusoidal functions on the web panel and web stiffeners (Fig. 13 (c)): 1) Local panel imperfections: Based on the first eigenmode from LBA under pure shear, with amplitude = $\min\{L_{panel}/200; h_w/200\}$; 2) Global panel imperfections: Half-wave sinusoidal shape applied to transverse, longitudinal and diagonal stiffeners, with amplitude =

$h_w/400$; 3) Local twist imperfections: Rotational imperfections on intermediate, longitudinal and diagonal stiffeners, with a maximum twist of $1/50$. A sensitivity analysis was conducted to assess the impact of imperfection shape and amplitude [89], using maximum amplitudes to ensure conservative resistance estimations.

5.2. Results and discussion

The numerical results were evaluated by examining the ultimate shear load attained in the simulations for the ten cases analysed. These cases included: U1, U3 – Benchmark cases with unstiffened webs with $\alpha=1$, $\alpha=3$; T1, T3 – transversely stiffened webs with $\alpha=1$, $\alpha=3$; L1, L3 – longitudinally stiffened webs with $\alpha=1$, 3; DT1, DT3 – diagonally stiffened webs along tension diagonal with $\alpha=1$, $\alpha=3$; DC1, DC3 – diagonally stiffened webs along the compression diagonal with $\alpha=1$, $\alpha=3$. Table 4 summarises the key results numerically obtained, including the shear buckling coefficient ($k_{\tau,FEM}$), elastic shear buckling load ($V_{cr,FEM}$) – obtained using the LBA under pure shear – and ultimate load ($V_{u,FEM}$) – resulting from the GMNIA. For reference the shear plastic resistance $V_{pl} = f_{y,w} h_w t_w / \sqrt{3}$ is shown.

By adopting an intermediate transverse stiffener which divides the web panel into two subpanels, the ultimate shear resistance increased 38% in the case of short web panels ($\alpha=1$) when compared to the unstiffened web case. The inclusion of a longitudinal stiffener in slender webs of square panels ($\alpha=1$) resulted in a 12% increase in ultimate load-carrying capacity. Furthermore, the use of a diagonal stiffener led to a more significant resistance increase, ranging from 28% to 39%, depending on whether the stiffener was oriented along the tension or compression diagonal, respectively. In the case of long web panels ($\alpha=3$), the effect of the intermediate transverse stiffener was reduced, yet improved the ultimate resistance by 27%. In contrast, the longitudinal stiffener became more significant compared to square panels, yielding an approximate 46% increase in ultimate shear load. On the other hand, when a diagonal stiffener was oriented along the tension diagonal in long panels, the improvement was relatively modest (15%). However, when the stiffener was aligned with the compression diagonal, a significant 66% increase in shear resistance was observed.

Fig. 14 presents the relative shear resistance, V_{FEM}/V_{pl} , plotted against the vertical displacement measured at the lower left corner of the web panel, normalized by the panel length, a . The benchmark response, represented by the unstiffened webs, is illustrated through cases U1 and U3.

Comparable ultimate shear resistances were achieved using either transverse stiffeners or diagonal stiffeners aligned with the compression diagonal, with both configurations attaining $V_{u,FEM}/V_{pl}$ ratios close to the plastic capacity of the web of 0.90 and 0.93, respectively, for short panels ($\alpha=1$). However, the transverse stiffeners induced larger vertical deformations in the girder and greater out-of-plane displacements of the web compared to the diagonal (C) stiffeners.

Furthermore, similar global responses – characterized by vertical and out-of-plane displacements – were observed when employing either longitudinal stiffeners or diagonal stiffeners aligned with the compression diagonal, regardless of panel length (short or long webs). Although diagonal stiffeners are longer and thus require more steel, their alignment with the compression diagonal resulted in superior ultimate shear resistance without increasing the vertical displacement of the plate

girder.

In contrast, stiffeners aligned with the tension diagonal also contributed to improved shear resistance, but at the expense of significantly greater vertical deformation to reach peak load. This configuration proved to be the least efficient, particularly for long web panels ($\alpha=3$).

Except for transversely stiffened webs in short panels (T1), all configurations achieved ultimate shear resistance before reaching the 5% strain threshold. Furthermore, the post-peak load response indicated that transversely stiffened webs experienced less reduction in load-carrying capacity compared to girders stiffened longitudinally or diagonally. This behaviour suggests that the latter configurations failed primarily due to the combined buckling of the web and stiffeners, despite the use of strong stiffeners (with $\gamma_{sl}=100$ or $\gamma_{sd}=100$). These findings imply that adopting closed-section stiffeners could offer a more effective solution in such cases. Fig. 15 shows the out-of-plane web displacement obtained at the ultimate load V_{FEM} for each case.

The use of a longitudinal stiffener, as well as the implementation of a diagonal stiffener along the compression diagonal, effectively reduced web out-of-plane displacement by approximately 40 and 42% for $\alpha=1$, and 42 and 33% for $\alpha=3$, respectively, when compared to the unstiffened web scenario, while also increasing the girder's ultimate load, as reported in Fig. 15 and Table 4. In contrast, the application of a transverse stiffener and diagonal stiffener aligned with the tension diagonal required greater vertical displacement of the girder to achieve its ultimate load (see Fig. 14 and 15). As a result, the corresponding out-of-plane web displacements at this stage exceeded those observed in the benchmark unstiffened configuration.

Table 5 compares the shear buckling resistances provided by the available key design proposals and code-based formulations, normalized to the obtained FE resistance. As noted by several researchers, the shear resistance model provided in EN 1993-1-5:2024 [11] underestimates the actual shear resistance of plate girders with slender and/or short web panels [33,34,63,89]. This conclusion was obtained for the case of transversely and the case of longitudinally stiffened webs. Moreover, the shear buckling predictions made by the models of Section 4 for diagonally stiffened square webs ($\alpha=1$) slightly overestimate the girder capacity – an exception being the model proposed by Yuan et al., which provided more accurate estimations for short panels. In the case of long panels ($\alpha=3$), the numerical resistance was higher than the model's predictions.

Fig. 16 shows the von Mises stress distribution obtained shortly after the shear resistance (V_{FEM}) is reached, as illustrated by the provided colour scale, capped at 355 MPa. The failure mode is highlighted, confirming that longitudinal and diagonal stiffeners reported out-of-stiffener plane deformations. Moreover, a noticeable widening of the tension diagonal stress band may be observed when additional web stiffeners are used in conjunction with the end post stiffeners.

Furthermore, the diagonal stiffener efficiently widens the tension band, thereby enhancing the shear resistance of the plate girder. However, the additional force transmitted along the tension diagonal may lead to increased loading on the intermediate transverse stiffeners. As a result, the interaction between multiple diagonally stiffened webs and intermediate transverse stiffeners should be further investigated within the context of continuous stiffened panels.

Table 5

Comparison of the shear buckling resistance using design formulations vs. the FE results.

Aspect ratio [-]	Unstiffened	Transverse	Longitudinal	Diagonal (C)		
	EN 1993-1-5	EN 1993-1-5	EN 1993-1-5	Jetteur	Hayatdavoodi	Yuan
$\alpha=1$	0.742	0.834	0.796	1.156	1.156	1.025
$\alpha=3$	0.868	0.797	0.758	0.757	0.757	0.840

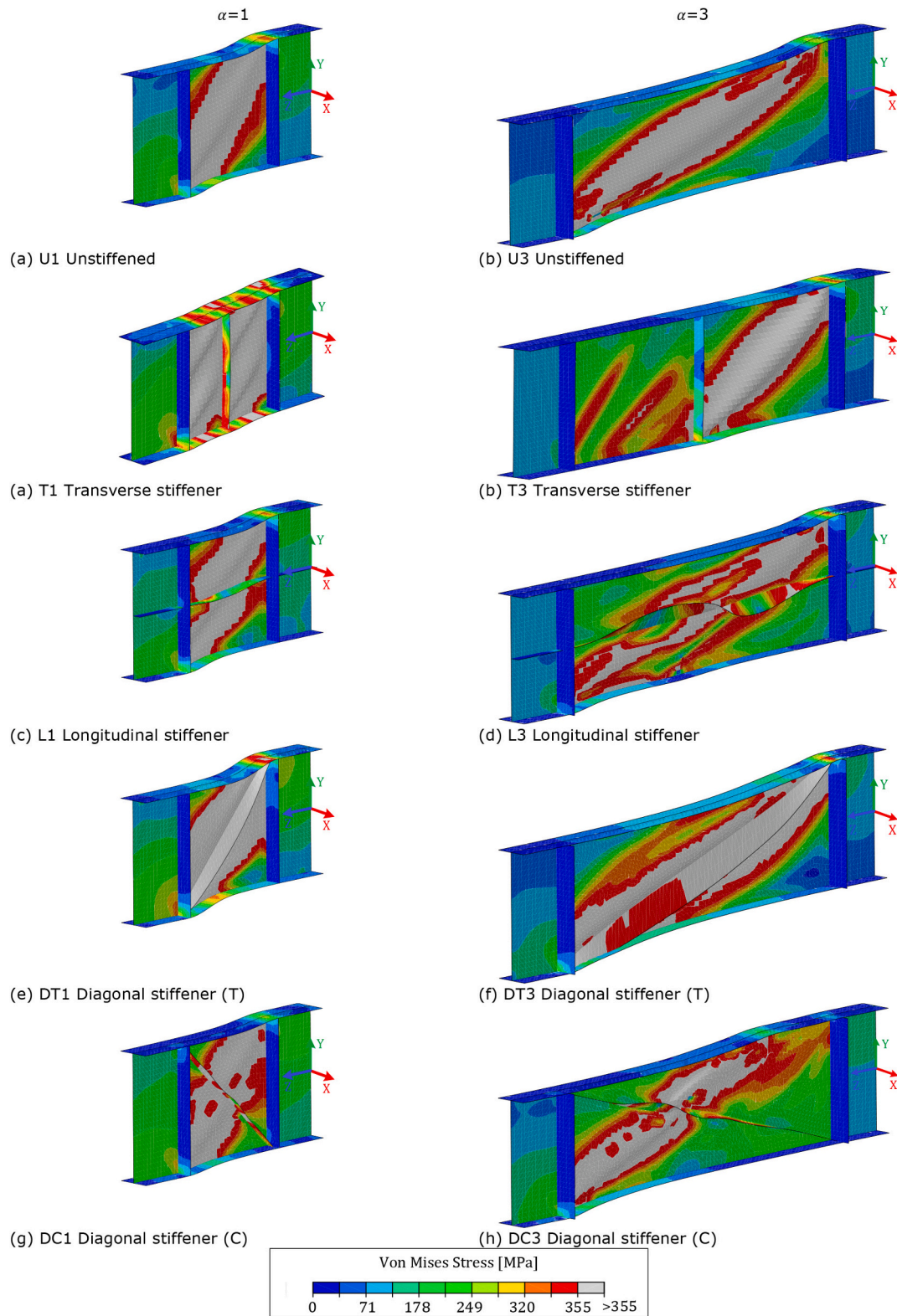


Fig. 16. FE von Mises stresses for a load stage after the maximum shear V_{FEM} for: (a) U1, (b) U3, (c) T1, (d) T3, (e) L1, (f) L3, (g) DT1, (h) DT3, (i) DC1, (j) DC3. Vertical disp. amplified 10x.

6. Conclusions

This paper presented a comprehensive review of the role of transverse, longitudinal, and diagonal web stiffeners in enhancing the shear

buckling resistance of I-shaped steel plate girders. Key findings and observations from literature and finite element (FE) simulations are summarized as follows:

1. Effect of web stiffeners on shear resistance

- Transverse stiffeners subdivide the web, increasing elastic shear buckling resistance and contributing to the out-of-plane rigidity of the web, which is essential for the development of the post-buckling tension field. These elements are particularly effective in short panels, although their rigidity must increase with decreasing web thickness or aspect ratio. Despite some conservatism, current design models, especially EN 1993-1-5, remain reasonably accurate, with FE results confirming shear resistance gains of up to 35% for short panels and 27% for longer ones.
- Longitudinal stiffeners complement transverse ones, improving shear buckling performance, particularly in slender and long panels. When placed at mid-height, they reduce out-of-plane deformations and help the web approach its plastic shear capacity. FE simulations show up to a 46% increase in shear resistance for long panels, with web displacements reduced by about 40%.
- Diagonal stiffeners, when placed along the compression diagonal, interrupt shear-induced deflection patterns and engage a truss-like mechanism with the flanges. They significantly increase both elastic and ultimate shear resistance – outperforming stiffeners aligned with the tension diagonal. Among the three main design models reviewed (Jetteur, Hayatdavoodi, and Yuan), simplifications such as full plastification or idealized boundary conditions limit their broader applicability, especially in carbon steel girders. FE results show up to a 66% gain in resistance for long panels.

2. Effectiveness of web stiffeners

The effectiveness of web stiffeners depends on their geometry (e.g. bending and torsional rigidity), material properties, and panel slenderness. Transverse stiffeners are most efficient in short panels; longitudinal ones in long panels; and diagonal stiffeners are most effective when oriented along the compression diagonal, especially in square or near-square panels. Closed sections or enhanced torsional rigidity can further improve performance.

3. Design methods and limitations

Several design models were reviewed. While strength-based approaches exist, recent developments favour rigidity-based criteria, such as those by Kim et al., which incorporate key parameters like slenderness and steel grade. This is reflected in AASHTO and AISC provisions. EN 1993-1-5 offers simpler limits (e.g. γ_s values) but provides limited guidance for longitudinal and diagonal stiffeners.

FE simulations and literature indicate that existing models often oversimplify stiffener interactions, neglect torsional rigidity, or fail to account for realistic boundary conditions – particularly for diagonal stiffeners. Some models rely heavily on numerical validation, with limited experimental support.

4. Final remarks

The finite element analysis (FEA) carried out in this study complements the literature review by illustrating the influence of different stiffener configurations and benchmarking their behaviour against existing design methods. Although not a broad parametric study, it provides a representative comparison that supports the observed trends and helps assess the conservatism of current approaches.

Overall, the findings confirm that existing design models offer acceptable estimates of shear buckling resistance for typical configurations, important challenges remain regarding stiffener interaction modelling, diagonal stiffener treatment, and rigidity requirements for ultimate resistance. Further refinement of design methods – supported by advanced simulations and experimental validation – will be essential to enhance the accuracy, reliability and efficiency of future plate girder designs.

Notation

a	Length of a stiffened web [mm]
A_f	Area of flanges [mm ²]
A_s	Area of stiffener [mm ²]
A_w	Area of web [mm ²]
b	Width of stiffened web [mm]
b_f	Width of flanges [mm]
b_s	Width of stiffener [mm]
$b_{s,f}$	Width of stiffener flange [mm]
$b_{s,i}$	Width of the inner part of stiffener [mm]
$b_{s,s}$	Width of the outer part of stiffener [mm]
$b_{w,eff}$	Effective width of web [mm]
c	Distance between flange plastic hinges [mm]
C_1	Material coefficient for hot-rolled steels [-]
C_2	Material coefficient for hot-rolled steels [-]
CG	Position of the centre of gravity [mm]
D	Web bending rigidity [Nmm]
e	Design eccentricity effective stiffener – web [mm]
E	Modulus of elasticity [N/mm ²]
E_{sh}	Strain hardening modulus for hot-rolled steels [N/mm ²]
f_{Cl,ε_u}	Stress for hot-rolled steels [N/mm ²]
$f_{y,f}$	Yielding stress of flanges [N/mm ²]
$f_{y,s}$	Yielding stress of stiffener [N/mm ²]
$f_{y,w}$	Yielding stress of web [N/mm ²]
G	Shear modulus [N/mm ²]
h_w	Clear depth of web between flanges [mm]
h_s	Height of stiffener [mm]
$h_{sl,i}$	Depth of web sub-panel i [mm]
I_P	Polar second moment of area [mm ⁴]
I_s	Second moment of area of stiffener [mm ⁴]
I_T	Saint Venant torsional constant [mm ⁴]
J	Elastic rigidity parameter for transverse stiffener according to AASHTO / AISC [-]
J_s	Torsion constant of transverse stiffener [mm ⁴]
k	Plate buckling factor of web [-]
k_s	Buckling factor of stiffener [-]
k_{yy}	Interaction factor [-]
k_τ	Shear buckling factor of web [-]
l	Element buckling length [mm]
L	Element length [mm]
M_{Ed}	Bending moment [kNm]
$M_{f,pl}$	Plastic bending moment resistance of flanges [kNm]
$M_{f,Rd}$	Design bending moment of resistance of flanges [kNm]
$M_{f,Rk}$	Characteristic bending moment of resistance of flanges [kNm]
M_{st}	Bending moment of transverse stiffener [kN]
M_y	Bending moment y-y [kNm]
$M_{y,Rk}$	Characteristic y-y bending moment [kNm]
N_{Rk}	Characteristic axial force [kN]
N_{st}	Axial force of transverse stiffener [kN]
N_x	Axial force x-x [kN]
$P_{pl,s}$	Plastic resistance of stiffener [kN]
r	Curvature of plate girder [°]
R	Minimum girder radius in the panel [mm]
R_h	Hybrid factor according to AASHTO [-]
s	Maximum between a and h_w [mm]
t_f	Thickness of flanges [mm]
t_s	Thickness of stiffener [mm]
$t_{s,f}$	Thickness of stiffener flange [mm]
t_w	Thickness of web [mm]
u_1	Out-of-web displacement [mm]
V, V_{Ed}	Shear force [kN]
V_b	Characteristic shear force [kN]
$V_{b,f}$	Characteristic flange shear force [kN]
$V_{b,w}$	Characteristic web shear force [kN]

V_{cr}	Elastic buckling shear load [kN]
V_d	Post-buckling shear force of diagonal stiffener [kN]
$V_{EN1993-1-5}$	Shear buckling resistance according to EN 1993-1-5 [kN]
V_m	Post-buckling shear force of web [kN]
V_{pl}	Plastic web shear force [kN]
V_t	Post-buckling shear force of flanges [kN]
V_u	Ultimate shear force [kN]
V_z	Shear force z-z [kN]
$W_{pl,f}$	Plastic section modulus of flange [mm ³]
Z	Curvature perimeter of girder [mm]
α	Aspect ratio of web panel [-]
β	Coefficient according to AASHTO and AISC [-]; Curvature correction factor for the longitudinal stiffener rigidity to AASHTO [-]
β_{sl}	Correction coefficient for longitudinal stiffeners [-]
γ_{M1}	Partial factor for buckling [-]
γ_s	Stiffener relative bending rigidity [-]
γ_{s1}	Stiffener relative bending rigidity for elastic design [-]
γ_{s2}	Stiffener relative bending rigidity for post-buckling web design [-]
δ_s	Stiffener relative axial rigidity [-]
ε	Yield ratio [-]; Strain [-]
ε_{lim}	Limit strain [-]
ε_{sh}	Strain hardening for hot-rolled steels [-]
ε_u	Ultimate strain [-]
ε_y	Yielding strain [-]
η	Strain hardening factor [-]
λ_w	Web slenderness [-]
θ_D	Inclination angle of diagonal line of the web panel [°]
θ_s	Stiffener relative torsional rigidity [-]
λ_s	Stiffener slenderness [-]
λ_w	Web slenderness [-]
$\bar{\lambda}_s$	Relative stiffener slenderness [-]
$\bar{\lambda}_w$	Relative web slenderness [-]
ν	Poisson ratio [-]
ρ_t	Parameter according to AASHTO and AISC [-]
σ_{cr}	Elastic buckling stress [N/mm ²]
$\sigma_{cr,s}$	Elastic buckling stress of stiffener [N/mm ²]
σ_M	Bending moment stress [N/mm ²]
σ_N	Normal stress [N/mm ²]
σ_s	Stress of stiffener [N/mm ²]
$\sigma_{s,c}$	Compression stress of stiffener [N/mm ²]
$\sigma_{s,t}$	Tension stress of stiffener [N/mm ²]
σ_t	Stress of tension diagonal [N/mm ²]
σ_u	Ultimate stress [N/mm ²]
τ_{cr}	Elastic shear buckling stress [N/mm ²]
τ_u	Ultimate shear stress [N/mm ²]
τ_v	Shear stress [N/mm ²]
τ_y	Shear yielding stress [N/mm ²]
φ	Inclination angle of the tension field [°]
ϕ	Taper ratio [°]
ϕ_f	resistance factor for flexure according to AASHTO [-]
χ_{LT}	Reduction coefficient for lateral buckling [-]
χ_w	Reduction factor for shear buckling [-]
χ_w	Reduction factor for shear buckling [-]
w_0	Initial imperfection amplitude [mm]
$w_{st,lim}$	Limit deflection of the transverse stiffener [mm]

CRedit authorship contribution statement

Sérgio Nascimento: Writing – original draft, Validation, Methodology, Investigation, Formal analysis. **Filip Ljubinković:** Writing – original draft, Validation, Methodology, Investigation, Formal analysis. **João P. Martins:** Writing – review & editing, Validation, Supervision. **José J. Oliveira Pedro:** Writing – review & editing, Validation,

Supervision.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Sérgio Nascimento reports financial support was provided by Portuguese Republic Government Ministry of Science Technology and Higher Education. Filip Ljubinković reports financial support was provided by Portuguese Republic Government Ministry of Science Technology and Higher Education. Joao Pedro Martins reports financial support was provided by Portuguese Republic Government Ministry of Science Technology and Higher Education. Jose Oliveira Pedro reports financial support was provided by Portuguese Republic Government Ministry of Science Technology and Higher Education. Filip Ljubinković reports financial support was provided by Foundation for Science and Technology. Joao Pedro Martins reports financial support was provided by Foundation for Science and Technology. Filip Ljubinković reports financial support was provided by Regional Operational Programme CENTRO2020. Joao Pedro Martins reports financial support was provided by Regional Operational Programme CENTRO2020. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors confirm that they have no known financial or personal conflicts that could have appeared to have an impact on the research presented in this paper.

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Data availability

Data will be made available on request.

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