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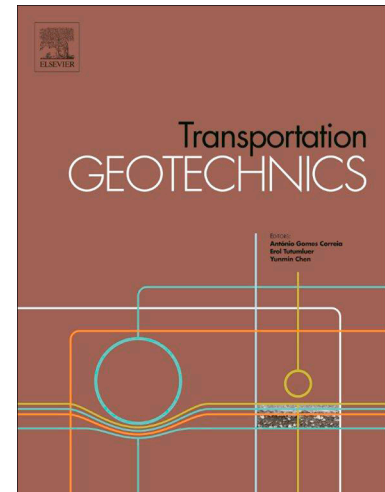
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LONG-TERM DEFORMATION PREDICTION OF A TROPICAL RAILWAY SUB-BALLAST SOIL IN HEAVY-HAUL TRACKS USING A HIGH-CYCLE ACCUMULATION MODEL

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Declaration of interests

- ☐ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
- ☒ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Guilherme Barbosa de Castro reports financial support was provided by Vale SA. Jonathan Saico reports financial support was provided by Vale SA. Rosangela Motta reports financial support was provided by Vale SA. Edson de Moura reports financial support was provided by Vale SA. Robson Costa reports financial support was provided by Vale SA. Liedi Bernucci reports financial support was provided by Vale SA. Andre Paixao reports financial support was provided by Vale SA. Eduardo Fortunato reports financial support was provided by Vale SA. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

HIGHLIGHTS

- Experimental and numerical study of a tropical lateritic sub-ballast soil.

- Application of the High-Cycle Accumulation (HCA) model in 3D FEM analysis.
- Calibration of HCA parameters using cyclic triaxial test data.
- Cement stabilization (3%) significantly improved long-term deformation resistance.
- Validated FEM-HCA framework for predicting heavy-haul railway substructure performance.

ABSTRACT:

The increase in axle loads in heavy-haul railways demands sub-ballast materials capable of maintaining long-term performance under severe cyclic loading, particularly in tropical environments. This study addresses the long-term deformation behaviour of tropical lateritic soils used as sub-ballast layers in heavy-haul railway tracks subjected to increased axle loads. The investigation focuses on untreated lateritic soil and lateritic soil treated with 3% cement under rehabilitation conditions using a Rail-mounted Formation Rehabilitation Machine (RFRM). The methodological framework combines laboratory testing and advanced numerical modelling. Mechanical characterization and cyclic triaxial tests were conducted to evaluate resilient behaviour and permanent deformation, while a 3D finite element model incorporating the High-Cycle Accumulation (HCA) approach was calibrated and applied to the untreated lateritic soil to assess long-term deformation trends under 32.5 t and 40 t axle loads. The results indicate good agreement between numerical predictions and laboratory measurements for the untreated soil, validating the adopted modelling strategy within the converged stress states. The untreated soil compacted with a vibratory plate (RFRM) showed excessive permanent deformation under 40 t/axle loading, demonstrating the limitations of this compaction method for heavy-haul conditions. In contrast, the 3% cement-treated soil significantly reduced deformation and ensured stable long-term behaviour in the laboratory tests under both load levels. Overall, the study confirms that low cement content treatment markedly improves the resilience of tropical lateritic sub-ballast soils and that the HCA-based numerical framework is a useful comparative tool for evaluating the nonlinear long-term performance of untreated lateritic sub-ballast. The findings provide practical guidance for the design and rehabilitation of heavy-haul railway infrastructure in tropical regions.

Keywords: Railway; Sub-ballast; Tropical soil; High-Cycle Accumulation model; Finite element modelling

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1 INTRODUCTION

The need for a more efficient and high-capacity transportation system has increased pressure on the railway sector to improve track quality and reduce travel time and maintenance. As noted by Raymond et al. (1976; 1978) and Ionescu et al. (1998), infrastructure is the most maintenance-intensive element, a trend also reflected in recent Brazilian investments (CNT, 2020). Developing more economical, efficient, and sustainable solutions, either through alternative materials or improved maintenance and rehabilitation methods, is therefore essential to reducing railway infrastructure costs, particularly for heavy-haul freight transport.

The railway track sub-ballast plays a key role in load distribution and subgrade protection against water infiltration. Conventionally, sub-ballast materials are required to meet strict gradation and hydromechanical criteria to ensure long-term stability under a high number of load cycles and large axle loads ($> 30 \times 10^6$ cycles and > 25 t/axle). However, in tropical regions, naturally occurring lateritic soil, although not meeting standard gradation specifications, can be considered as alternative materials when properly designed and assessed under cyclic loading. In this context, the sub-ballast layer is known to contribute significantly to track durability and to the reduction of maintenance and rehabilitation costs (Selig and Waters, 1994; Li et al., 2016).

This issue is particularly relevant for the Carajás Railway (EFC), a Brazilian heavy-haul railway dedicated mainly to iron ore transport, which is currently transitioning from axle loads of 32.5 t to 40 t. In the present rehabilitation context, the sub-ballast layer is one of the key structural layers being redesigned and reinforced, and the possible use of locally available lateritic soils as sub-ballast material is attractive because it can reduce haulage distances and construction costs. This approach is also supported by positive experience in road engineering applications (Indraratna and Nulalaya, 1991; Bernucci, 1995; Nogami and Villibor, 1991, 1995; Villibor et al., 2009) and by more recent studies indicating the feasibility of lateritic soils in railway sub-ballast layers (Guimarães et al., 2021; Castro et al., 2022, 2025; Costa et al., 2022). To improve construction efficiency and productivity, rehabilitation works on heavy-haul railways are increasingly expected to rely on Rail-mounted Formation Rehabilitation Machines (RFRM), which allow *in situ* reuse of existing materials and rapid reconstruction of the track formation. However, the performance of RFRM technology is strongly dependent on the compaction efficiency achieved during operation, which may represent a limitation when applied to fine-grained or lateritic soils.

The RFRM excavates the existing track bed, blends it with alternative materials (such as recycled ballast or local soil), and subsequently compacts the mixture using a vibratory plate system (Mundrey, 2009; Schilder and Piereder, 2000; Auer et al., 2008; Fu et al., 2017). However, its effectiveness in compacting tropical lateritic soils remains uncertain, particularly under more demanding heavy-haul conditions, since the equipment was originally developed for conventional European railway tracks operating at axle loads of up to 25 t. Furthermore, the reliance on vibratory plate compaction raises concerns, as vibratory energy is generally more effective for granular sands, whereas cohesive or fine-grained soils are better compacted using kneading mechanisms such as sheepfoot rollers (Lambe and Whitman, 1969; ASTM D4253-16). Massad (2016) also observed that kneading compaction tends to produce denser, less

permeable, and more homogeneous soil structures, especially when compacted slightly above the optimum moisture content.

As axle loads increase, such as in the case of the EFC, compaction quality and material performance become even more critical. Castro et al. (2025) demonstrated that RFRM vibratory plates are inefficient in compacting fine and granular lateritic tropical soils, resulting in reduced short-term mechanical performance compared to roller compaction in terms of soil moduli. However, while short-term properties such as resilient modulus may suggest acceptable performance, ensuring long-term performance of these materials requires assessing additional factors (e.g., gradation, moisture, density, void ratio, layer thickness, stress state), including permanent deformation under repeated loading. Thus, parameters such as elastic modulus, resilient modulus and California Bearing Ratio alone may not be sufficient to predict long-term behaviour of railway substructure materials.

Traditional cyclic triaxial laboratory test equipment have been applied largely to evaluate the permanent strain of geomaterials for pavement purposes through different stress states (deviatoric and confining stresses), not considering the PSAR but always horizontal or vertical (Lekarp and Dawson, 1998; Werkmeister, 2003; Wichtmann, 2005; Niemunis et al., 2005; Werkmeister et al., 2005; Sas et al., 2017; Gu et al., 2020; Duan et al., 2023). As a result, advanced laboratory tests such as a Modified Torsional Multi-ring Shear apparatus and Multiaxial Direct Simple Shear tests (DSS) have also been developed to evaluate the influence of PSAR on cumulative plastic deformation in rail track subgrade materials and granular sandy materials, indicating that single-point cyclic loading tests may underestimate actual deformation characteristics (Wichtmann, 2005; Niemunis et al., 2005; Gallage et al., 2016; Dareeju et al., 2017; Ramos et al., 2020). Discrete Element Method (DEM) and hollow cylinder tests have been used to investigate the effects of stress path and rotation (moving wheel load) on granular materials behaviour, underscoring the importance of multiaxial stress conditions to not underestimate particle rearrangements, degradation and settlement predictions (Thevakumar, 2018; Bian et al., 2020; Ramos et al., 2020; Indraratna et al., 2024). Long-term performance of railway substructure under cyclic loading has been also investigated, including full-scale performance assessments under repeated traffic loads with Principal Stress Axis Rotation (PSAR) applying different actuators placed in different positions to simulate train passage showing the importance of this phenomenon on accumulated strain of the railway track geomaterials (Cebasek et al., 2018; Esen et al., 2023).

Recent systematic reviews have highlighted the limitations of traditional empirical methods obtained through laboratory and field tests and advocated advanced numerical approaches for predicting resilient and permanent deformation in subgrade and granular materials, accurately (Yang et al., 2025). That is why, the analysis of the long-term deformation of the railway substructure materials have also been supported by advanced numerical modelling developed in the last decades (Abdelkrim et al., 2003; Niemunis et al., 2005; Wichtmann et al., 2015; Wichtmann, 2016; Ling et al., 2017; Varandas et al., 2020; Paixão et al., 2021). Varandas et al. (2020) and Paixão et al. (2021) have also incorporated the PSAR on their numerical modelling to predict the long-term deformation of ballast and sub-ballast under train passage. Although the importance of principal stress axis rotation (PSAR) in controlling long-term deformation behaviour is well recognized, conventional cyclic triaxial tests remain highly valuable due to their practicality, repeatability, and efficiency in terms of testing time and data interpretation. When properly designed and interpreted, these tests provide reliable and precise estimates of material behaviour under cyclic loading, particularly for comparative assessments and parameter calibration in engineering applications (Wichtmann, 2016; Varandas et al., 2020; Ramos et al., 2020; Paixão et al., 2021).

Regarding the numerical modelling of the nonlinear long-term deformation behaviour of unbound granular materials subjected to cyclic loads, previous research has used elastoplastic, shakedown theory and mechanistic-empirical approaches (Lekarp et al., 2000; Werkmeister et al., 2001; Monismith et al., 1975; Guimarães, 2009; Sun et al., 2017). Nevertheless, conventional elastoplastic models are generally inadequate for simulating the response of materials subjected to a very high number of load cycles, as typically encountered in railway infrastructure. These models often yield inaccurate results, require high computational effort, and are prone to numerical instability (Niemunis et al., 2005; Ling et al., 2017; Ramos et al., 2020). Moreover, they were originally developed to simulate individual load-unload cycles and to capture behaviour near failure, whereas railway track geomaterials are expected to operate predominantly within the elastic or near-elastic regime. They are limited to low cycle conditions unlike railway scenarios with millions of cycles, becoming difficult to implement and to achieve computational accuracy of the results, while mechanistic-empirical models use results from triaxial tests and they do not simulate the stress paths and PSAR induced by train passage (Ramos et al., 2020).

To overcome this limitations, advanced High-Cycle Accumulation (HCA) models have been proposed to better simulate shakedown behaviour and permanent deformation under cyclic and moving loading (Suiker and De Borst, 2003; Niemunis et al., 2005; François et al., 2009; Pastén et al., 2014). The Cyclic Densification Model by Suiker and De Borst (2003), based on the Drucker-Prager yield surface, predicts residual deformation and effectively accounts for the maximum stress state. It distinguishes between deviatoric and volumetric components (i.e. frictional sliding and volumetric compaction) and does not explicitly separate static and cyclic components of the loading. Unlike Suiker and De Borst (2003), François et al. (2009) clearly distinguish between the static and cyclic components of loading, and the cyclic component is considered small relative to the static one. The model focuses on deformation accumulation in granular soils under low-amplitude cyclic or dynamic loading to simulate nearby traffic of vibrating equipment incorporating the effects of mean stress, cyclic amplitude, and void ratio (i.e., initial density). Varandas et al. (2020) and Paixão et al. (2021) implemented a 3D finite element model combining this approach with the hypoelastic $k-\theta$ model. The Bochum model (Niemunis et al., 2005), implemented in ABAQUS commercial software (UMAT - a User Material Model, Fortran-coded, subroutines that calculate the constitutive equations at the Gauss point level on every iteration), incorporates anisotropy, void ratio evolution, and loading direction changes, and has been validated for up to 2 million cycles (Wichtmann, 2005; Wichtmann et al., 2009; Wichtmann and Triantafyllidis, 2015), though it requires extensive calibration (Varandas et al., 2020). While these models have shown good predictive capability for conventional unbound granular materials, their calibration and application to tropical lateritic soils, particularly for railway sub-ballast layers subjected to heavy-haul loading, remain scarce. This lack of calibration represents a significant gap in current research, given the proven strength, durability, and water resistance of lateritic soils in tropical environments.

To mitigate compaction inefficiency, improve workability, control expansiveness, enhance water resistance and improve long-term performance, chemical treatment using low cement contents has been proposed as a viable alternative (Ceratti, 1991; ABCP, 2004). Several studies have demonstrated that cement contents between 2% and 4% significantly enhance the hydromechanical behaviour of soils used in pavement structures (Guthrie et al., 2002; Miller et al., 2006; Abu-Farsakh et al., 2015; Beja et al., 2020; Solihu, 2020; Amhadi and Assad, 2021; Valoura et al., 2022; Iyaruk et al., 2022; Peng et al., 2023). These improvements are particularly pronounced in lateritic sandy soils, due to chemical interactions between cement particles and clay minerals. Research on cement-treated lateritic soils is synthesized in Consoli et al. (2021) and Suebsuk et al. (2023). However, despite these benefits, the nonlinear long-term behaviour of cement-treated lateritic soils for sub-ballast under heavy cyclic railway loading has not yet

been sufficiently investigated. In addition, cement treatment changes the material response from essentially unbound to partially bonded or semi-rigid, so mechanisms such as fatigue or shrinkage cracking may become relevant and should be assessed separately when establishing final design recommendations.

Despite the extensive research on railway track substructures, limited attention has been given to the long-term deformation behaviour of tropical lateritic sub-ballast soils under heavy-haul loading conditions, particularly when compacted using Rail-mounted Formation Rehabilitation Machines (RFRM). In addition, the applicability of advanced HCA modelling frameworks to these materials remains largely unexplored. Therefore, the main objective of this study is to investigate the long-term cyclic deformation behaviour of a tropical lateritic sub-ballast soil through an integrated approach combining cyclic triaxial testing of untreated and cement-treated materials with 3D finite element modelling of the untreated lateritic soil under axle loads of 32.5 t and 40 t. The results are intended to support more informed design and rehabilitation decisions for railway substructures in tropical regions, while also clarifying the scope and limitations of the adopted HCA-based numerical framework. The methodological sequence adopted in this study is shown in Figure 1.

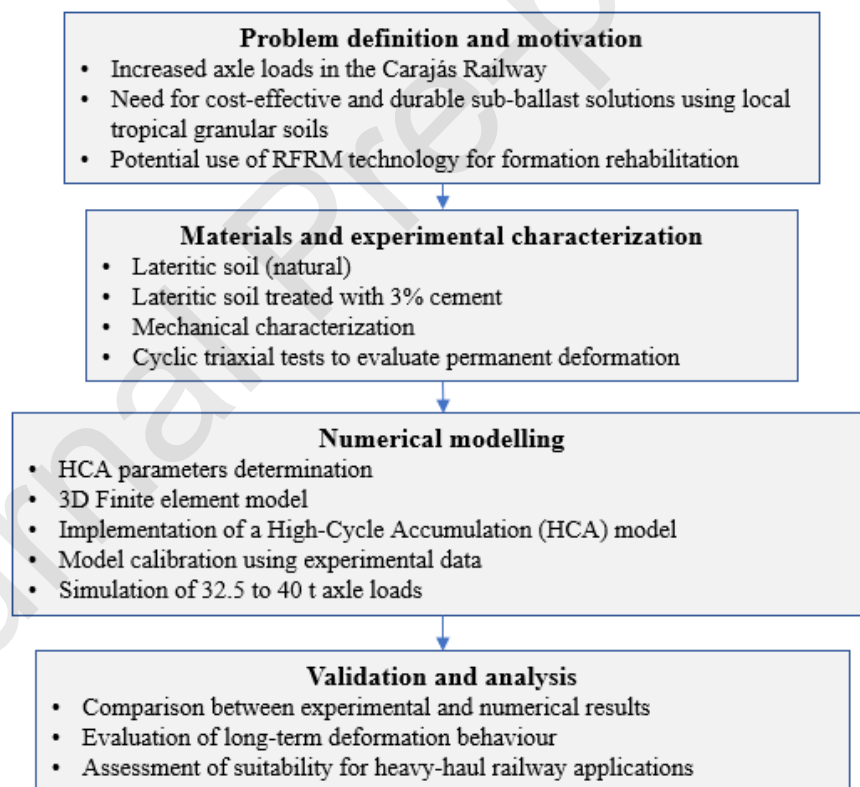


Figure 1. Research flowchart of the present study

2 MATERIALS AND METHODS

2.1 Materials

Two tropical soils were selected for this study. Preliminary analyses indicated that one soil presented suitable characteristics for potential use as sub-ballast material (lateritic soil), while the other showed lower bearing capacity and was therefore adopted as a reference

subgrade material (non-lateritic silty-sand soil) for comparison purposes. Figure 2 shows the sub-ballast and subgrade soils.



Figure 2. Studied soils: (a) sub-ballast soil; (b) subgrade soil

2.1.1 Laboratory characterization

Regarding the detailed physical characterization of the studied materials, it can be found in Castro et al. (2025). According to MCT (Miniature Compacted Tropical soil method) and HRB (AASHTO method) classifications, the lateritic soil is classified as a well-graded gravelly sandy lateritic soil, characterized by a broad particle size distribution (favourable HRB A-2-4 and MCT LA-LA' classifications), low plasticity ($PI = 8\%$), and the presence of aggregated particles bonded by fine lateritic fractions rich in iron and aluminium oxides. These features typically confer high stiffness (resilient modulus > 200 MPa at standard Proctor energy), good drainage capacity, and favourable mechanical performance under cyclic loading when adequately compacted, being suitable for sub-ballast applications due to its mechanical stabilisation and compliance with AASHTO Grading Range C (Guimarães et al., 2021).

In contrast, the second soil corresponds to a non-lateritic silty-sand soil classified as A-7-5 according to the AASHTO system. It presents a high fines content with moderate plasticity ($PI = 12\%$) and lacks the aggregated and cemented fabric typical of lateritic soils, resulting in lower stiffness and greater susceptibility to moisture variations and permanent deformation under cyclic loading. Table 1 and Figure 3 show the key physical and index properties of the soils and the three gradation ranges (e.g., Ranges I, II, and III) commonly specified for granular materials used in road pavement base and sub-base layers. In railway systems, these layers are functionally equivalent to the sub-ballast layer, which provides load distribution, improves track stiffness, and protects the subgrade from excessive stresses and moisture-related degradation. These ranges represent minimum and maximum gradation envelopes adopted in road engineering standards and are also frequently applied to railway sub-ballast classification (SUP-00048).

Table 1. Physical characteristics of the soils

Sample/Material		
	Silty-sand soil	Lateritic soil
Properties/Characteristics		
Passing No. 4 sieve (%)	95.5	52.6
Passing No. 10 sieve (%)	85.9	34.1
Passing No. 200 sieve (%)	38.3	9.0
Optimum moisture content - w_{opt} (%)	17.0	12.2
Maximum dry density - $\gamma_{d,max}$ (kg/m ³)	1685	1944
Liquid Limit - LL (%)	41	25
Plasticity Limit - PL (%)	29	17
Plasticity Index - PI (%)	12	8
HRB soil classification method	A-7-5 (clayey)	A-2-4 (silty or clayey gravel and sand)
¹ Mini-Compacted-Tropical soil classification method (Large Disk Method)	Non-Lateritic. Silty/Sandy	Lateritic sandy

¹Godoy and Bernucci (2000)

2.1.2 Material mix design process

Previous studies, such as Castro et al. (2025), demonstrated the limited efficiency of compacting fine-grained silt-clay soils using vibration, as typically applied in RFRM operations. These authors evaluated the compaction efficiency of a standard RFRM vibratory plate compactor through laboratory box tests and also investigated the potential improvement in workability, strength, and compaction when local soils were chemically treated with cement. Thus, a soil-cement mixture was designed following DNIT 142 (2022), ET-DE-P00/004 (2024) and ET-DE-P00/056 (2024), which applies to base and sub-base layers in road pavements (Figure 3). A 3% type II Portland cement content was adopted for cost-effective and sustainable soil-treatment, supported by studies such as Valoura et al. (2022) and Peng et al. (2023).

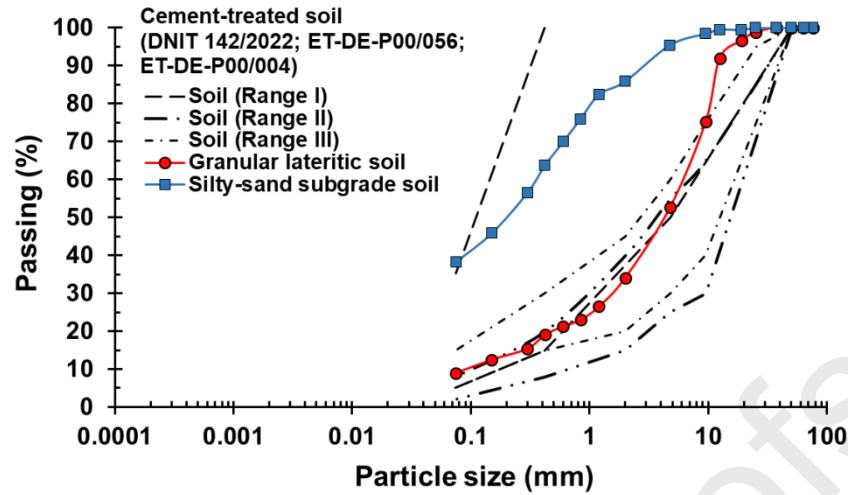


Figure 3. Cement-treated soil grain size distribution criteria for pavement base and sub-base layers (ET-DE-P00/004; ET-DE-P00/056; DNIT 142/2022)

2.2 Cyclic triaxial tests - Resilient modulus and permanent deformation

The mechanical characterization of the soils was performed through cyclic load triaxial tests on cylindrical specimens (0.10 m diameter, by 0.20 m high) to determine resilient modulus values, following DNIT 134-ME (2023) standard which was based on AASHTO T 307-99 (1999).

Firstly, the tested materials were homogenized at the specified moisture content, placed in a hermetically sealed plastic bag and stored in a humidity-controlled chamber for a minimum period of 12 hours prior to compaction. The chemically treated materials were compacted right after mixing and homogenized with cement. Standard Proctor compaction energy was adopted as a reference to prepare the specimens following EN 13286-2 (2010), as recommended by Schilder and Piereder (2000) and Auer et al. (2008), based on their field and lab studies conducted in Austria focusing on the RFRM application. Thus, specimens were compacted under Standard Proctor as well as at lower energy levels, simulating the vibratory plate compaction and the degree of compaction (D_{Pr}) of 86% applied and obtained by Castro et al. (2025) in box test experiments.

The mechanical characterization of the soils was performed through cyclic load triaxial tests on cylindrical specimens (0.10 m in diameter and 0.20 m in height) to determine resilient modulus values, following the DNIT 134-ME (2023) standard, which was developed based on the AASHTO T 307-99 (1999) procedure. Although the stress levels defined by AASHTO were not applied in this study, they are presented in Table 2 for reference and comparison purposes. A loading frequency of 1 Hz is adopted for the resilient modulus test and a conditioning phase with 500 cycles prior to testing. The test setup also included two displacement transducers (LVDTs) placed 180° apart to measure vertical deformations, applying stress conditions and amplitudes representative of base layers in pavements. The resilient modulus results were calculated by the formulation of Pezo et al. (1993) with the correlation parameters k_1 , k_2 and k_3 and stress states (σ_d and σ_3) from laboratory data, as shown in Equation (1) and Table 2.

$$\text{Resilient modulus } (E) = k_1 \sigma_3^{k_2} \sigma_d^{k_3} \quad (1)$$

Table 2. Stress state - resilient modulus test

DNIT 134-ME (2023) - applied AASHTO T 307-99 (1999)			
σ_3 (kPa)	σ_d (kPa)	σ_3 (kPa)	σ_d (kPa)
20	20	20.7	18.6
	40	20.7	37.3
	60	20.7	55.9
35	35	34.5	31.0
	70	34.5	62.0
	105	34.5	93.1
50	50	68.9	62.0
	100	68.9	124.1
	150	68.9	186.1
70	70	103.4	62.0
	140	103.4	93.1
	210	103.4	186.1
105	105	137.9	93.1
	210	137.9	124.1
	315	137.9	248.2

	140	-	-
140	280	-	-
	420	-	-

σ_d = deviatoric stress; σ_3 = confining stress

Permanent deformation tests were performed to evaluate the accumulated (plastic) strains in the specimens, based on measurements obtained from sensors installed in the actuator or piston of the Material Testing System (MTS) press. The mechanical behaviour of the materials in terms of permanent deformation was analysed in accordance with the Brazilian test instruction DNIT 179 (2023), which establishes procedures for cyclic triaxial testing of tropical soils under repeated loading to evaluate plastic strain accumulation and material stability. This standard follows principles similar to international methodologies for permanent deformation assessment, such as those defined in AASHTO and EN standards. Additionally, the methodology developed in studies by Wichtmann (2005; 2016) was applied to predict the accumulation employing the HCA model as explained in the next sections.

For the permanent deformation test, the recommended loading frequency is 2 Hz, although frequencies ranging from 1 to 5 Hz may also be used. Thus, a loading frequency of 5 Hz was applied in the permanent deformation tests in accordance with DNIT 179 (2023). Regarding the HCA model, Wichtmann (2016) did not consider loading frequency to be a determining factor, noting that most studies in the literature, as well as his own research (Wichtmann, 2005; Wichtmann et al., 2010), did not find any dependency between loading frequency (up to 8 Hz) and the rate of permanent deformation accumulation in granular materials.

The long-term behaviour tests were performed with 100,000 to 150,000 load cycles. The number of cycles was defined considering the HCA model calibration Method (b) proposed by Wichtmann (2016), the recommendations of the DNIT 179 (2023) standard, and the observed material response until deformation stabilisation (shakedown). The HCA model calibration process Method (b) proposed by Wichtmann (2016) and applied in this research assumes that a single cyclic triaxial test may be used to determine some HCA parameters C_{N1} , C_{N2} , and C_{N3} , while others C_{amp} , C_e , C_p , and C_y can be estimated from grain size distribution or physical index such as average diameter (d_{50}) and uniformity coefficient (C_u) or minimum void ratio (e_{min}). Alternatively, three cyclic triaxial tests with specimens at different initial densities can also be performed to obtain a relevant model parameter. These procedures are recommended as the minimum acceptable approach when the number of cyclic tests must be limited, and they are considered only slightly less accurate than the robust and highly laborious test protocols that comprise with at least 11 cyclic triaxial tests with varying stress state, average mean pressure, frequency, density, and amplitudes (Wichtmann et al., 2015).

Regarding stress states, those most representative of the sub-ballast layer were assumed for the materials, based on stress results in the railway track obtained through field instrumentation and monitoring (Indraratna et al., 2011a; McHenry et al., 2015; Costa et al., 2017; 2022) and numerical modelling developed in this study which is described later. Table 3

and Table 4 summarize the stress state conditions and the different cyclic test settings performed to evaluate granular soil with degree of compaction of 100% and 86%, and the latter treated with 3% of cement.

Table 3. Stress state - permanent deformation test (DNIT 179, 2023)

σ_3 (kPa)	σ_d (kPa)	$*(\sigma_d + \sigma_3)/\sigma_3$
40	40	2
40	80	3
40	120	4
40	160	5

$*(\sigma_d + \sigma_3) = \sigma_1$; σ_d = deviatoric stress; σ_3 = confining stress

Table 4. Applied stress configuration for different materials and test conditions - permanent deformation test

Material	Test number	$\sigma_{d0,min}$ (kPa)	$\sigma_{1,ave}$ (kPa)	σ_3 (kPa)	σ_d (kPa)	q^{amp}	$\sigma_{d,max}$ (kPa)	$\sigma_{d,min}$ (kPa)	Initial condition
									Confining state
									$\sigma_1, \sigma_2, \sigma_3$
Granular soil ($D_{Pr} \approx 100\%$)	1	13	66.4	40	40	14	40	13	40
	2	13	86.4	40	80	34	80	13	40
	3	21	110.4	40	120	50	120	21	40
	4	13	127.2	40	162	74	162	13	40
	5	40	140.0	40	160	20	120	40	40
	6	80	240.0	80	240	80	240	80	80
	7	120	360.0	120	360	120	360	120	120
Granular soil ($D_{Pr} \approx 86\%$)	1	13	66.4	40	40	14	40	13	40
	2	13	86.4	40	80	34	80	13	40
	3	13	106.4	40	120	54	120	13	40
	4	13	126.7	40	160	75	161	13	40
Granular soil ($D_{Pr} \approx 86\%$) + cement	1	13	66.4	40	40	14	40	13	40
	2	13	86.4	40	80	34	80	13	40
	3	13	106.4	40	120	54	120	13	40
	4	13	126.4	40	160	74	160	13	40

$\sigma_{d0,min}$ = initial deviatoric stress; $\sigma_{1,ave}$ = average vertical stress; q^{amp} = deviatoric stress amplitude; CEM = cement

2.3 Nonlinear and long-term behaviour analysis of the railway track

2.3.1 Model description - geometry, boundary and load conditions

A three-dimensional (3D) finite element model was developed using the ABAQUS software, widely validated in technical and scientific studies (Alves et al., 2020; Pereira et al., 2023; Castro et al., 2022, 2025). The primary purpose of this model was to simulate the resilient and nonlinear long-term behaviour, in terms of stresses, resilient and plastic deformations, of the sub-ballast material under dynamic loading conditions. The main constitutive model applied was the HCA model, implemented through a UMAT code provided by Niemunis et al. (2005).

This approach was based on the research conducted by Paixão et al. (2021) and Varandas et al. (2022) that adapted Suiker and De Borst (2003) and Niemunis et al. (2005) research to simulate cyclic behaviour of railway infrastructures. Given that the focus of this study is the evaluation of sub-ballast material, the proposed model included: (i) the wagon wheel loads applied directly to the rails, without considering interaction with the wagon spring system; (ii) the superstructure components (rails, elastic pads, sleepers) and the ballast layer; and (iii) the infrastructure layers (sub-ballast and subgrade).

Boundary conditions were defined as follows: vertical planes normal to the z-axis (longitudinal) and x-axis (transversal) were fixed in x and z directions and restrained in rotation about x and y axes except for vertical movement ($U_y \neq 0$). For simplicity, a rectangular-section beam was used to represent the rail, in accordance with Paixão and Fortunato (2010) and Indraratna et al. (2011b), with geometry equivalent to TR-68 rails (67.4 kg/m and 0.186 m high, moment of inertia $I_{xx} = 3920.9 \text{ cm}^4$, and base and head section moduli of 462.1 cm^3 and 388.4 cm^3 , respectively), suitable for a $1600 \pm 8 \text{ mm}$ broad gauge. A high-density polyethylene (HDPE) rail pad (density $\approx 1.10 \text{ g/cm}^3$) was modelled beneath the rail to transmit stresses to the sleeper. The rail base was assumed perfectly bonded to the elastic pads, which in turn were also perfectly bonded to the sleepers. This assumption is justified by the preload applied by the fastening system and considering that there is no slip between these components, eliminating the need for additional contact elements.

The sleepers were modelled as 51 trapezoidal prisms, representing prestressed Monoblock concrete sleepers spaced 0.60 m apart, with symmetry considered (i.e., the model was reduced by half, assuming realistic boundary conditions along the divided surface, to improve computational efficiency without compromising the accuracy of the results). Each sleeper had a length of 2.80 m, width of 0.30 and 0.21 m on the bottom and the top, respectively, height of 0.23 m, mass of 417.47 kg, and density of 2.495 g/cm^3 . The interaction between the sleeper base and the ballast surface was modelled with surface-to-surface contact elements incorporating normal and frictional behaviour, allowing vertical separation of adjacent sleepers under load. The penalty method was used for both normal and tangential contact formulations with the stiffness assumed as default as it is calculated automatically by the software based on the model properties, with friction coefficient of 0.80 calibrated via trial-and-error to ensure convergence and acceptable sleeper penetration ($< 0.1 \text{ mm}$). The small-sliding formulation was

considered adequate, and the friction coefficient assumed was in accordance with other studies Hosseini and Esmaeili (2016), Santana and Estaire (2019), Navaratnarajah et al. (2022).

All superstructure elements were defined in accordance with Brazilian freight railway technical standards, including 80-DES-000A-18-8000 (2009), 80-EM-030A-58-8016 (2012), PIM 01 (2015), PIM 10 (2015), SUP-00026 (2023), and SUP-00028 (2023).

The ballast and sub-ballast layers, each 0.30 m thick, were modelled as granular materials. At the ballast-sub-ballast interface, a friction coefficient of 0.5 was used, as recommended by Indraratna et al. (2013), applying the penalty method without allowing separation. The subgrade was assumed to be a well-compacted soil layer with a depth of 5 m and constant resilient modulus, following the same approach as other FEM studies (Shan et al., 2013; Varandas et al., 2014; Paixão et al., 2015; Varandas et al., 2020; Wang et al., 2023; Pereira et al., 2023). In the numerical model, the ballast and subgrade resilient behaviour were represented as linear elastic with constant resilient modulus obtained in Castro et al. (2025), whereas the permanent deformation behaviour was simulated only for the untreated lateritic sub-ballast through the nonlinear HCA formulation calibrated from cyclic triaxial tests. This modelling choice was adopted to focus the long-term constitutive nonlinearity on the layer of interest and to keep the analyses computationally tractable. Therefore, the model should be interpreted as a comparative framework for untreated lateritic sub-ballast response rather than as a full constitutive description of all track layers or of cement-treated bonded materials. Loading conditions simulated wagons with axle loads of 32.5 t and 40 t, composed of two bogies and four axles per wagon (GDU type), operating at an average speed of 80 km/h, as specified in SUP-00026 (2023). All materials were considered homogeneous and isotropic.

Regarding the modelling steps and applied load process, they can be divided into, based on Niemunis et al. (2005) (Figure 4):

- a) Calculate the initial stress state resulting from self-weight and monotonic loading using a conventional plasticity model. Gravitational effects were included in the first analysis step named as Geostatic step, representing the self-weight of components to define initial stress conditions, ensuring equilibrium and convergence of the numerical model. The Geostatic initial step was then activated in the model with the components own weight calculated in each top of its underneath layer using a conventional (e.g. multi-surface plasticity model).
- b) Calculate at least the first two cycles applying an extended hypoplastic model, implicitly. The next steps involve implicitly calculating at least the first two load cycles by applying the extended hypoplasticity model developed by Niemunis and Herle (1997), which accounts for the nonlinearity of the soil. These cycles can be represented either as a sinusoidal function with periodic variation in load amplitude or as a linear ramp load. The first strain amplitude is disregarded due to its high and irregular magnitude (non-uniform cycle). In contrast, the second strain amplitude is calculated in the same manner and designated as the Recording Cycle.
- c) Record the logarithmic strain path during the first regular cycle at each integration point. Evaluate the tensorial strain amplitude from the recorded strain path, if it remains constant until it is recalculated during a control cycle.
- d) Determine the rate of accumulation. Once the recording cycle and the corresponding strain amplitude are defined, the accumulation of plastic strain is calculated as the number of load cycles increases, a process referred to as the pseudo-creep phase.
- e) After a certain number of cycles, variations in density and stress state alter the strain amplitude. This is subsequently updated during another phase, known as the control cycle.

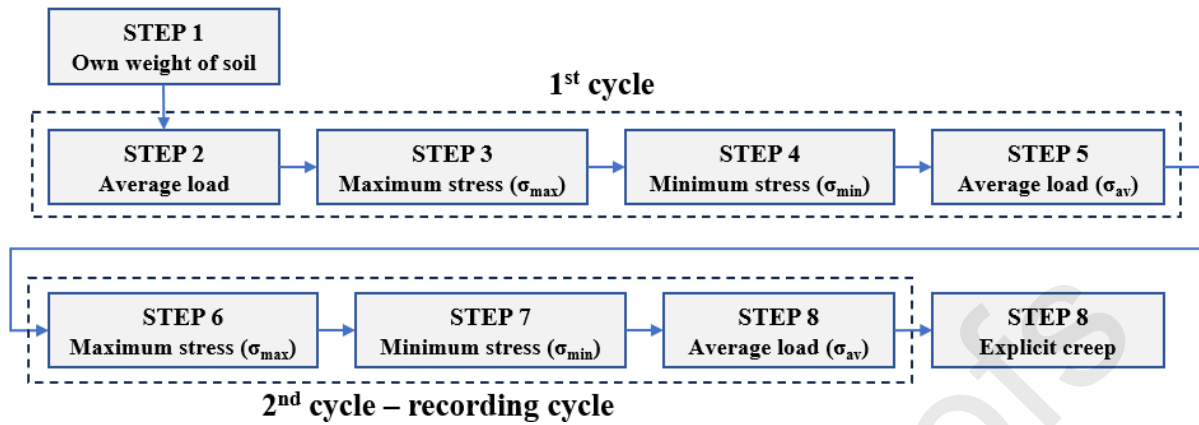


Figure 4. Schematic representation of the modelling steps

The equivalent cyclic loads were applied at a single location for simplification. In other words, it was not simulated the translational movement of a moving train in a specific direction. The cyclic loading is simulated with the axles kept in fixed positions, as depicted in Figure 6, rather than representing the continuous passage of axles along the model. This approach was assumed given the high computational effort required and the difficulty in achieving convergence in the modelling processes for the long-term behaviour predictions of the sub-ballast layer. However, it neglects stress path rotation and load migration along the track, which may lead to an underestimation of accumulated permanent deformation. Despite moving vehicles may be another practical example of the HCA model application, it was not part of the scope of this research.

There is also the challenge for reproducing the multi-dimensional nature of stress or strain paths in soils in a laboratory environment for calibration purposes as highlighted in Niemunis et al. (2005). Wichtmann (2005) and Niemunis et al. (2005) reported that the well representation of the change of shape and polarization (i.e., orientation) of the cycle in the strain space may increase the rate of accumulation, based on an extensive laboratory testing program with drained cyclic triaxial and multiaxial DSS tests. The laboratory cyclic triaxial tests used for material characterization and model calibration presented further in this research, apply only repeated axial loading under constant confining pressure through cyclic triaxial tests and therefore cannot reproduce the translation of principal stress axes induced by train passage. Consequently, the characteristic non-proportional, heart-shaped stress path observed in railway track materials under moving loads cannot be obtained. Despite these limitations, cyclic triaxial testing remains established method for evaluating permanent deformation and calibrating HCA models as shown in Werkmeister et al. (2001), Fortunato et al. (2012), Paixão et al. (2021) and Mota et al. (2024).

Regarding the application of cyclic loads, loads frequencies were applied in a sinusoidal form with periodic variation in load amplitude, represented by the Equations (2) and (3). Vertical loads in the form of ascending and descending ramps, where the load magnitude varies linearly as a function of the step time can also be applied as an alternative to the sinusoidal load as shown by the schematic representation of the applied loads in Figure 5.

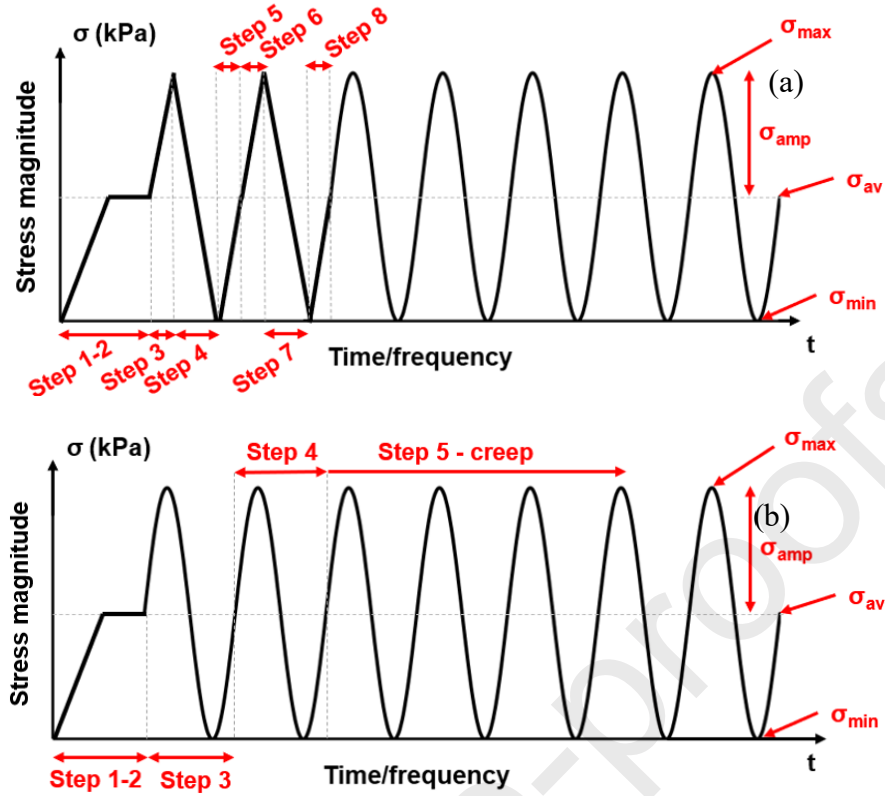


Figure 5. Schematic representation of the applied loads: (a) linear ramp; (b) periodic functions

$$a = A_0 + \sum_{n=1}^N [A_n \times \cos n\omega(t - t_0) + B_n \times \sin n\omega(t - t_0)] \text{ for } t \geq t_0 \quad (2)$$

$$a = A_0 \text{ for } t < t_0 \quad (3)$$

where a is the load amplitude (kN), A_0 is the initial load amplitude (kN), N is the number of load cycles (kN), ω is the circular frequency, and t_0 is the starting time.

Although a detailed discussion on mesh sensitivity and numerical characteristics is provided in later sections, it is important to note that multiple levels of mesh refinement were tested. The final model was selected based on a balance between computational efficiency and result accuracy. The number of degrees of freedom, finite elements, and nodes considered as key indicators of the model's scale and numerical complexity are discussed later in the next topics and shown preliminary in the developed model in Figure 6.

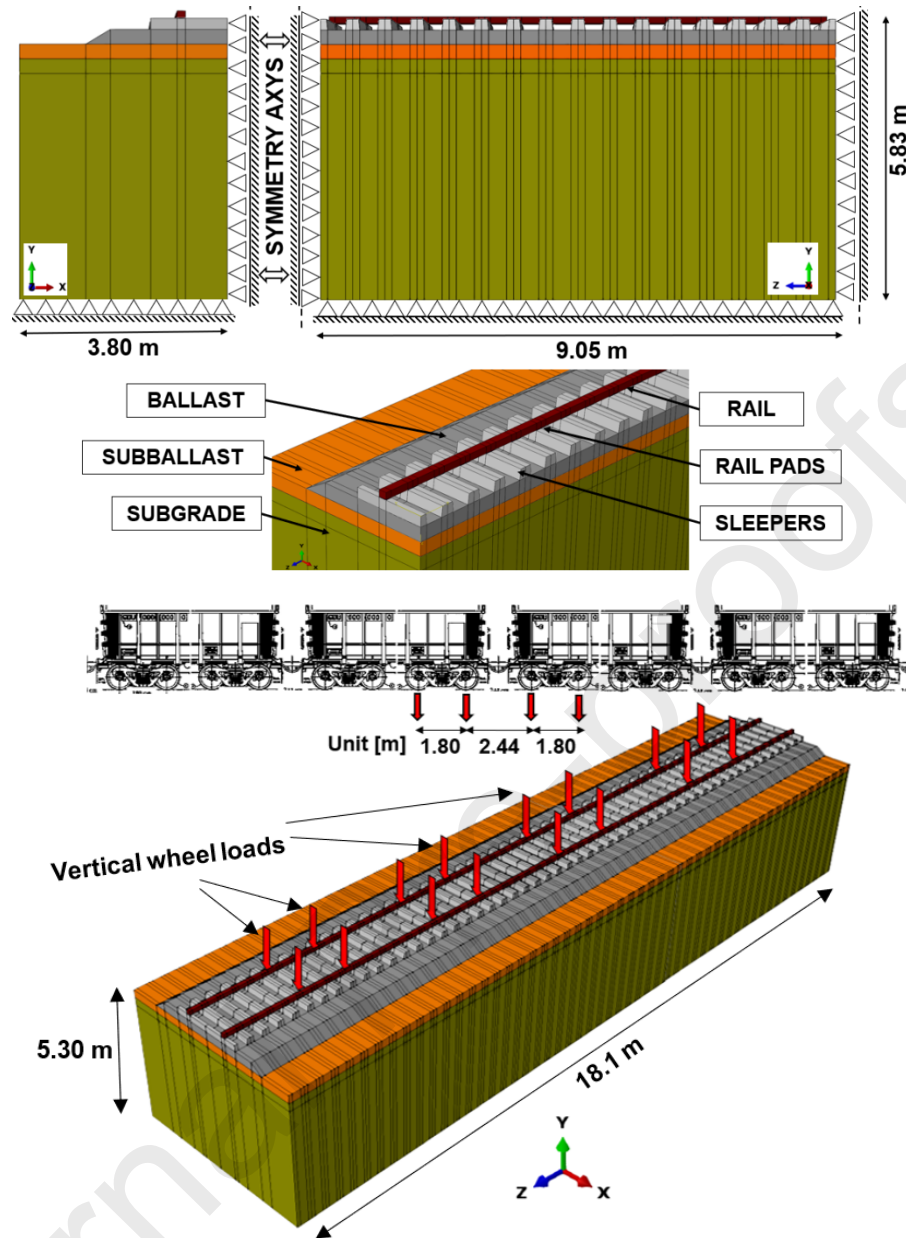


Figure 6. Developed 3D railway track model and load configuration

The process of applying cyclic loads was divided into different steps to represent the conditions of 32.5 and 40 t/axle, respectively, as shown previously in the schemes of Figure 4 and Figure 5: (i) an average load of 23.8 and 29 t/axle; (ii) a maximum load of 45.5 and 56 t/axle, respectively, due to the application of a dynamic increment factor of 1.4, obtained through field measurements at EFC by Castro et al. (2025); (iii) a minimum load of 2 t/axle, assumed based on near-zero values obtained from load cells used for monitoring a railway track in the field by Costa et al. (2022); and (iv) a load amplitude of 21.8 and 27 t. Table 5 presents the load configurations applied on the HCA model.

Table 5. Cyclic load application data - HCA model input

Railway track modelling - Cyclic load application input

Scenario (t/axle)	Axle load (kN)			Wheel load (kN)			Applied wheel load (kN)	
	Average	Maximum	Minimum	Average	Maximum	Minimum	Average	Amplitude
32.5	238	455	20	118.8	227.5	10	118.8	108.8
40.0	290	560	20	145.0	280.0	10	145.0	135.0

Considering that the greater the number of analysis points in the model, the longer the computational time required to process the results, the selection of analysis points was based on the material to be analysed and the assumed critical locations where relevant deformations and displacements occur in the material and not in the whole model. For example, it was selected for numerical computation only the sub-ballast layer. Regarding the vertical strain (ϵ_{yy}) result in the sub-ballast layer, it was firstly obtained the strains at the centroid (i.e., arithmetic average of 8 integration points) of each of the 36 elements located in the block under the sleeper edge, and adjacent to the rail edge. Secondly, the resulting strain in the block was calculated by the weighted average of the centroid strains, based on the different weight or volume of each element. Finally, the resulted strain in the block was assumed as the accumulated strain at the sub-ballast layer. This methodology was applied considering that the accumulated strain of the sub-ballast layer is influenced by all the elements over its thickness, and not only by the surface elements. Regarding the vertical displacement results (U_{yy}), they were computed underneath the rail edge at the sub-ballast top and rail bottom, assuming that this location is more impacted from the traffic loads as in Paixão et al. (2021). Figure 7 shows the detailed scheme of the analysed locations.

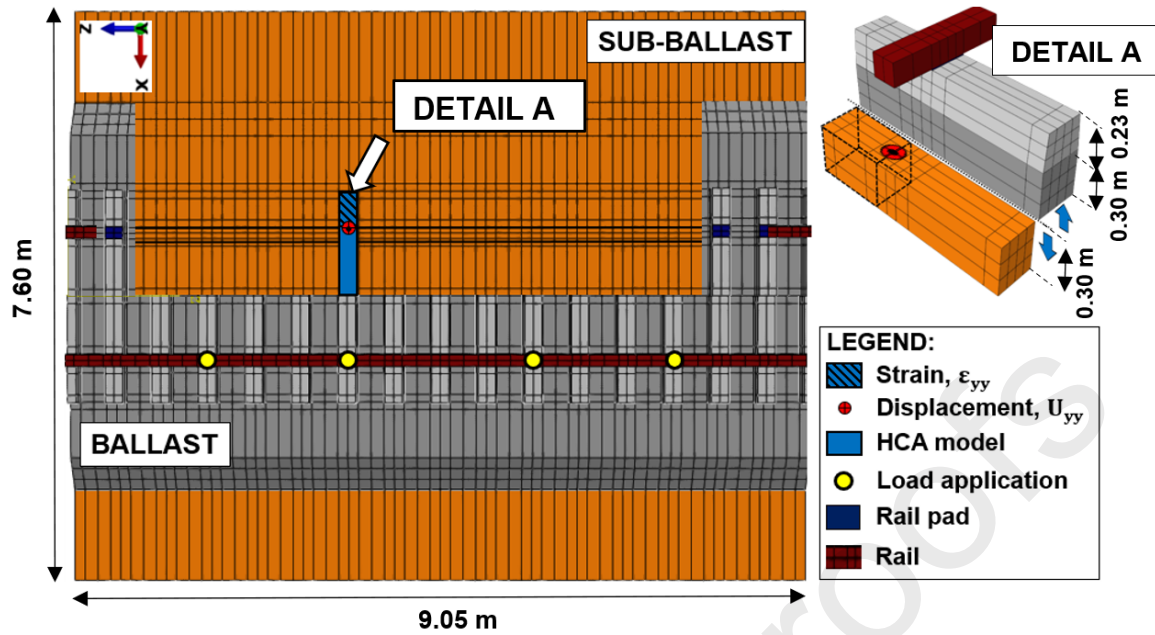


Figure 7. Schematic representation of the location of the evaluated points (plan view with portions of the track omitted to show the sub-ballast layer)

2.3.2 Meshing configuration and load frequency analysis

The verification of the finite element mesh was conducted by analysing the impact of the variation in the refinement of the mesh (and corresponding number of elements) on the convergence of displacement and deformation results, as well as their differences and the computational time required for each analysis. The objective was to improve the reliability and efficiency of the model, in terms of computational effort, without compromising its accuracy. Model 1 comprises 1,280 elements, whereas Model 6 consists of 52,696 elements. Each node of the elements possesses three translational degrees of freedom. First-order solid, continuous and structured hexagonal/sweep wedge shape block elements with eight nodes (C3D8 type) were employed, enabling the computation of translations and rotations through linear interpolation.

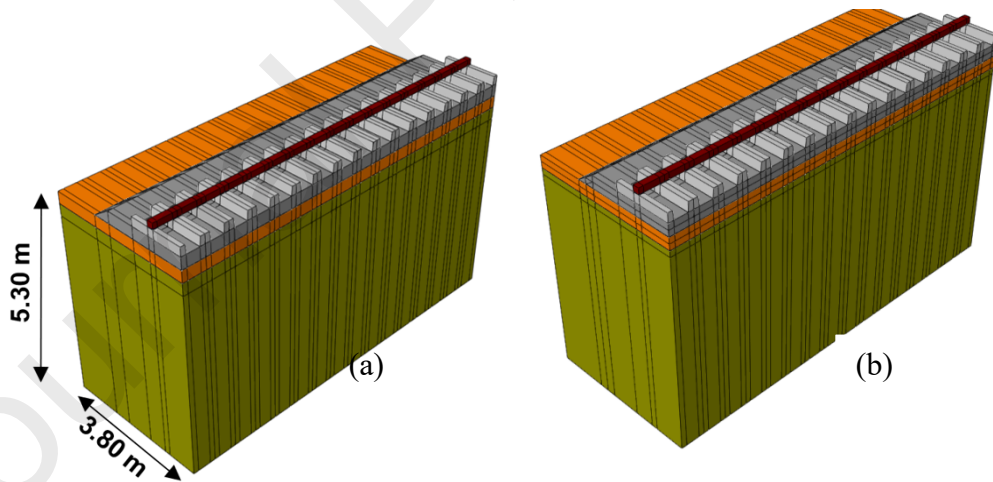
The selected loading frequencies between 1 and 8 Hz were adopted based on both experimental and operational considerations. Wichtmann (2005) demonstrated that loading frequency within this range has negligible influence on the permanent strain accumulation rate in granular materials. Additionally, the loading frequency can be estimated from the train speed and axle spacing, since each axle passage generates one load cycle at a fixed point. For a typical freight train travelling at 80 km/h (22.2 m/s) with an axle spacing of approximately 2.40 m, the corresponding loading frequency is about 9.25 Hz. Therefore, the adopted frequency range is representative of realistic field conditions and suitable for simulating heavy-haul railway loading scenarios. As already mentioned in the last topic, symmetry was introduced into the 3D model in both the transverse and longitudinal directions in order to reduce computational time without compromising its representativeness. The represented model also contains a reduced number of sleepers due to the implementation of the symmetry conditions. Table 6 and Figure 8 detail the considered material's parameters and the different finite element meshes tested in

the numerical model, ranging from the coarsest mesh (Mesh 1) to the most refined mesh (Mesh 6), used to evaluate mesh sensitivity and ensure numerical accuracy.

Table 6. Mechanical parameters and geometric characteristics simulated

Material	Parameter				
	Resilient modulus (MPa)	Stiffness (kN/mm)	Poisson's ratio	Density (g/cm ³)	Thickness (m)
Rail	210000	-	0.30	7.860	-
Elastic/rigid rail pad	26/134	100/500	0.01	1.150	0.006
Sleeper	32000	-	0.25	2.500	-
Ballast	208	-	0.20	1.730	0.300
Sub-ballast (lateritic soil)		*		2.150	0.300
Subgrade (silty-sand soil)	70	-	0.30	1.800	5.000

*See Table 12 and 12 for the input data assumed for the sub-ballast



(c)

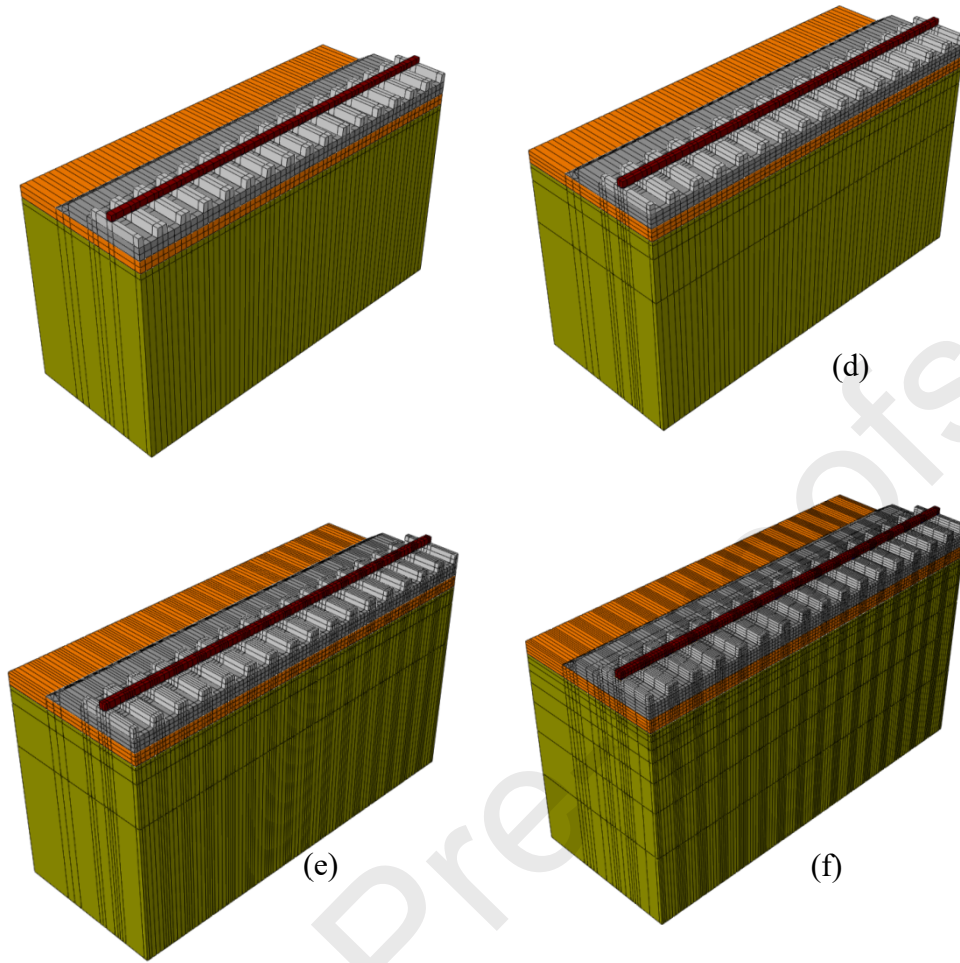


Figure 8. Finite element meshes tested for the developed numerical model with increasing mesh refinement: (a) Mesh 1 - coarsest mesh; (b) Mesh 2; (c) Mesh 3; (d) Mesh 4; (e) Mesh 5; and (f) Mesh 6 - finest mesh.

2.3.3 UMAT verification and testing

The validation and calibration process of the proposed railway material HCA model was performed by an axisymmetric analysis simulating cyclic triaxial tests with a 0.10 m diameter by 0.10 m high sample, using data parameters from Lukkezen (2016). The accumulated permanent deformation for 1×10^5 cycles obtained in this research applying Abaqus/CAE were compared to those obtained by Plaxis implementation on Lukkezen (2016). Four operation modes (e.g., implicit mode, recording mode, explicit mode, and control mode) were inserted, applying sinusoidal periodic and linear ramp axial loads as illustrated by Figure 9. The numerical evaluation was performed for 1×10^5 load cycles, deviatoric stress of $\sigma_d = 60$ kPa, deviatoric + confining stress or $\sigma_1 = 300$ kPa, and confining stress of $\sigma_3 = 150$ kPa.

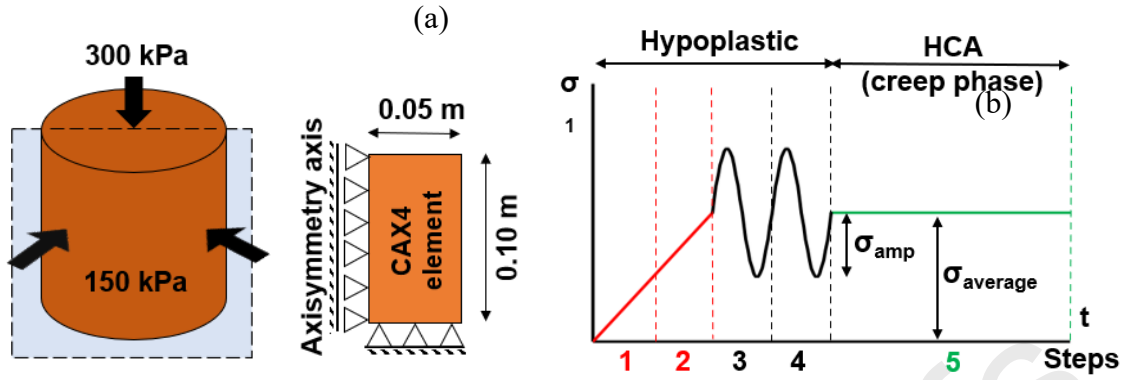


Figure 9. Axisymmetric analysis: (a) model conceptualization; (b) load configuration.

2.3.4 HCA railway track model parameters calibration

This study applies the HCA multivariable non-linear model developed by Niemunis et al. (2005) to predict the accumulation rate of granular materials through four independent functions, each representing the influence of parameters such as number of cycles, strain amplitude, mean pressure, average stress ratio, and void ratio. Niemunis et al. (2005) described and simplified the parameter determination process by isolating these functions and conducting laboratory tests. Equation (4) presents the accumulation rate formula.

$$\dot{\epsilon}^{acc}(N) = \dot{\epsilon}^{acc} \times \mathbf{m} = f_{amp} \times f_p \times f_Y \times f_e \times \dot{f}_N \times f_{\pi} \times \mathbf{m} \quad (4)$$

Where $\dot{\epsilon}^{acc}$ is the accumulated strain, $\mathbf{m}$ is the direction of accumulation, f_{amp} is the strain amplitude ϵ^{amp} function, f_p (kPa) is the average mean pressure p^{av} function, f_Y (kPa) is the average stress ratio η^{av} function, f_e is the void ratio e function, $\dot{f}_N$ is the number of cycles N function, and f_{π} is the change of polarization of the strain loop function.

In Equation (4), the accumulation rate of permanent strain ($\dot{\epsilon}^{acc}$) is expressed as the product of independent functions, each representing the influence of a specific physical mechanism. The vector $\mathbf{m}$ defines the direction of strain accumulation in deviatoric and volumetric components and depends mainly on the average stress ratio. The functions f_{amp} , f_p , f_Y , and f_e represent the influence of strain amplitude (ϵ^{amp}), average mean stress (p^{av}), average stress ratio ($\eta^{av} = q/p$), and void ratio (e), respectively. The function $\dot{f}_N$ describes the reduction in accumulation rate with increasing number of load cycles (N), based on a logarithmic-linear relationship. The function f_{π} represents the influence of cycle polarization or orientation; however, for conventional cyclic triaxial loading with constant stress direction, this factor may be taken as equal to unity ($f_{\pi} = 1$), as recommended by Niemunis et al. (2005) and Wichtmann et al. (2010, 2016). In this formulation, each function f represents a dimensionless scaling

function rather than a material constant, allowing the model to independently capture the effects of stress state, density, strain amplitude, and loading history on strain accumulation.

The $C_{\#}$ parameters $C_{amp}, C_p, C_Y, C_e, C_{N1}, C_{N2}$ and C_{N3} that influence on the intensity of strain accumulation $\dot{\epsilon}^{acc}$ are directly related to the f_{amp}, f_p, f_Y, f_e , and f_N functions, respectively, as detailed in Wichtmann (2005). Regarding the f_{π} function, Niemunis et al. (2005) introduced a non-physical state variable, the tensor π (back polarization), to account for cycle orientation effects. While changes in polarization can increase the accumulation rate, Wichtmann et al. (2015) demonstrated that the factor f_{π} can be omitted in practice, even with frequent polarization shifts. Accordingly, setting $f_{\pi}=1$ is sufficient, as confirmed by Wichtmann et al. (2010), who successfully predicted permanent deformations in pavements using the HCA model under constant cycle polarization. The critical friction angle (φ_{cc}) is applied to account for the direction of strain accumulation related to tensor $\mathbf{m}$.

The calibration of the HCA model parameters (e.g., $C_{amp}, C_e, C_p, C_Y, C_{N1}, C_{N2}, C_{N3}$) for the soil was carried out through laboratory tests and by assuming values obtained in the literature for sandy materials with similar grain size distribution. Among the three procedures recommended by Wichtmann (2016) for determining these parameters and calibrating them, the second method or improved simplified method was chosen, due to its simplicity. This method combines correlations with granulometric properties and a limited number of drained cyclic triaxial tests. While this procedure is recommended as a minimum standard for practical applications, it should be noted that conventional cyclic triaxial tests do not reproduce several key features of railway loading, such as the shape and polarization of strain loops (out-of-phase cycles), multi-axial stress conditions, and rotation of principal strain axes. These aspects are known to influence strain accumulation but are not captured in the present laboratory programme nor explicitly represented in the adopted HCA formulation.

Table 7 shows the basic physical index and particle size distribution data of the material applied for the calibration process. The parameter $C_{amp}=1.7$ was obtained in Wichtmann et al. (2009; 2015) as the mean value obtained through correlation with the indexes d_{50} and C_u recommended for a simplified calibration and assumed in this research.

Table 7. Physical index data for HCA parameter calibration of granular soil

Physical index and characteristics		Result
Compaction - Standard Proctor reference	Minimum void ratio - e_{min}	0.460
	Porosity - η	0.315
	Maximum dry density - $\gamma_{d,max}$ (g/cm ³)	1.944
	Optimum moisture content - w_{opt} (%)	12.2
Grain size distribution characteristics	Passing No. 4 sieve (%)	52.6

Passing No. 10 sieve (%)	34.1
Passing No. 200 sieve (%)	9.0
Effective particle size - d_{10} (mm)	0.09
Particle diameter at 30% passing - d_{30} (mm)	9.6
Median particle size - d_{50} (mm)	4
Particle diameter at 60% passing - d_{60} (mm)	6
Uniformity coefficient - C_u	67
Curvature coefficient - C_c	171

The minimum void ratio was obtained through samples preparation by experimental tests, such as Proctor compaction test. Regarding the maximum void ratio, it was determined by deposition of moist samples into a cylindrical reservoir with known mass and volume ($10.263 \times 10^{-6} \text{ m}^3$) avoiding external perturbation such as shaking or vibration of the reservoir. Table 8 shows the calculated density index applied as input in the HCA model.

Table 8. Calculated density index

Degree of compaction (based on Standard Proctor)	Assumed density index	Void ratio of the compacted sample	Maximum void ratio	Minimum void ratio
D_{Pr}	I_D	e	e_{max}	e_{min}
100%	1.00	0.46 (e_{min})	1.408	0.460
86%	0.86	0.60	1.408	0.460

It was assumed that the compaction condition at the optimum moisture content could provide the minimum void ratio and that the free deposition of the soil in a reservoir would reach the maximum void ratio. These approaches were based on ABNT NBR 16843 (2020) and ABNT NBR 16840 (2020), equivalent to ASTM D4253-16 and ASTM D4254-16.

Furthermore, it was considered that the degree of compaction (D_{Pr}) and density index (I_D) were equivalent (e.g., $D_{Pr} \approx 86\%$, and $I_D = 0.86$). This assumption was made because this relationship was not the focus of this research, as it varies according to soil type, specific gravity, grain size distribution and mineralogy (Boyer, 2019).

The parameters C_{N1} , C_{N2} and C_{N3} were determined based on the results of obtained from permanent deformation behaviour through cyclic load triaxial tests (i.e., deviatoric strain data). The following stress state conditions were simulated in these tests, based on stress state scenarios verified for the railway track by Costa et al. (2022), such as maximum vertical deviatoric stress of 100 kPa at the sub-ballast layer, and transversal stresses of 47 and 60 kPa underlying the sleeper-ballast and sleeper edge, respectively: (i) 40/40 kPa; (ii) 40/80 kPa; (iii) 40/120 kPa; (iv) 40/160 kPa.

It is important to highlight that radial deformation was not measured but only the deviatoric component of the axial deformation evaluated through the axial LVDTs. Thus, the deviatoric strain data was fitted to the function $\dot{\epsilon}_N$ as recommended by Wichtmann (2016). The maximum number of cycle applications evaluated in the calibration was 1×10^5 . The parameters C_{N1} , C_{N2} and C_{N3} were calibrated through a structured iterative process based on finite element simulations of cyclic triaxial element tests. Despite requiring the resolution of numerical convergence issues, this approach yielded a well-validated parameter set consistent with observed soil behaviour. The calibrated results were compared to the calibrated parameters obtained for a well-graded unbound granular pavement base material (UGM) studied by Wichtmann et al. (2010), based on its grain size distribution similarity as shown in Figure 10.

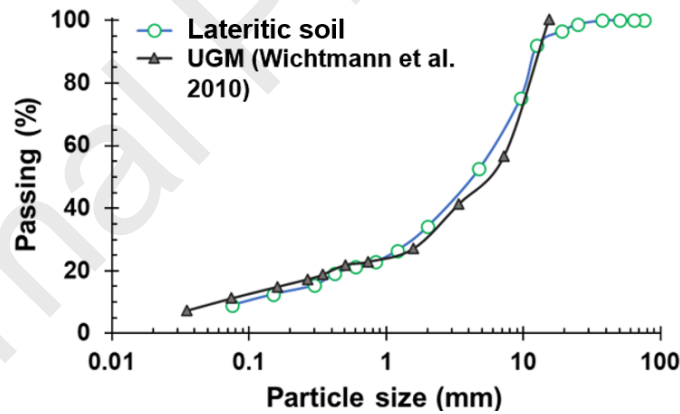


Figure 10. Comparison between the grain size distribution of lateritic soil and UGM material studied by Wichtmann et al. (2010)

Although the UGM was compacted under modified Proctor energy, typically achieving higher density, lower void ratios, and greater stiffness than standard Proctor energy, thereby enhancing higher load-bearing capacity and resistance to permanent deformation, the material constants C_{N1} , C_{N2} and C_{N3} capture the effects of cyclic preloading or cyclic history. These parameters are related to the rate of permanent deformation accumulation, determining how quickly or slowly the material compacts depending on the initial void ratio. Nevertheless, the overall shape of accumulation curve (ϵ^{acc}) remains essentially the same for dense or compacted samples, regardless the way the specimen has been previously preloaded or compacted, as

reported by Wichtmann (2016). That is why different compacted materials with similar grain size distribution could be compared to each other in terms of these parameters.

Regarding the extended hypoplastic material parameters applied in the implicit calculations, it was assumed the same parameters considered by Niemunis et al. (2005), Wichtmann (2005) and Lukkezen (2016) for $R_{\max}$, χ and β_R . The remaining parameters h_s , α , β , m_R , and m_T were determined through a rigorous iterative calibration process, combining systematic trial-and-error refinement with sound engineering judgement, and in Niemunis and Herle (1997), Niemunis et al. (2005), Wichtmann (2005) and Lukkezen (2016) for sandy materials. In contrast, the critical friction angle (ϕ_c), the initial void ratios corresponding to the minimum (e_{d0}), maximum (e_{i0}) and critical (e_{c0}), as well as Bauer's coefficient for proportional compression (n) were obtained via established correlations using the equations previously presented in Wichtmann et al. (2015). The parameters $R_{\max}$, χ , and β_R have control in the evolution of intergranular strain, governing small-strain stiffness, stiffness degradation, and the influence of loading direction in the hypoplastic model. According to Lukkezen (2016), these latter do not have a significant contribution to the total accumulation, being considered in the first accommodation cycles in the beginning of the accumulation to quantify the grain movements and interlocking. Wichtmann (2005) reported that the granular hardness parameter h_s controls the pressure-dependent stiffness of the material and represents the resistance of the granular skeleton to compression. the granular hardness h_s and exponent n define the pressure-dependent reduction of the void ratios e_i , e_c , e_d , and e as the mean pressure p increases. The constant α governs the influence of density on the peak friction angle ϕ_p . The constant β controls the increase in the objective stress rate $\dot{T}$ as the density of the specimen increases under constant D (strain rate). The values m_R and m_T control the volumetric and deviatoric stiffness of the material, respectively, governing its response under isotropic compression and shear loading. Niemunis and Herle (1997). Details about the hypoplastic constitutive model and its parameters can be found in Niemunis and Herle (1997) and Wichtmann (2005).

3 RESULTS AND DISCUSSIONS

3.1 Resilient behaviour of soils

Table 9 and Figure 11 summarize the resilient modulus results obtained through cyclic triaxial tests performed for the studied materials for different degrees of compaction and different stress levels.

Table 9. Resilient modulus results for the different materials

Material	D_{Pr}	Curing time	Resilient Modulus	
			(for $\sigma_d = 100$ kPa and $\sigma_3 = 40$ kPa)	(for $\sigma_d = 100$ kPa and $\sigma_3 = 100$ kPa)
	(%)	Days (d)	MPa	

			47	73
Silty-sand soil	≈ 100	Without cement treatment	53	80
			44	62
			121	229
	≈ 86		132	234
Lateritic soil		Without cement treatment	172	262
			179	263
	≈ 100		179	261
		2 (2d)	345	416
	≈ 86			
		3 (3d)	447	613
			525	670
	≈ 86	7 (7d)	389	562
			527	671
Lateritic soil + cement	≈ 86	28 (28d)	436	614
		3 (3d)	636	919
	≈ 100		651	918
			725	1003
	≈ 100	7 (7d)	654	928
			680	905

		738	1016
$\approx$ 100	28 (28d)	715	968
		762	1019

D_{Pr} - degree of compaction, σ_d, σ_3 - deviatoric and confining stresses at the sub-ballast due to train passage

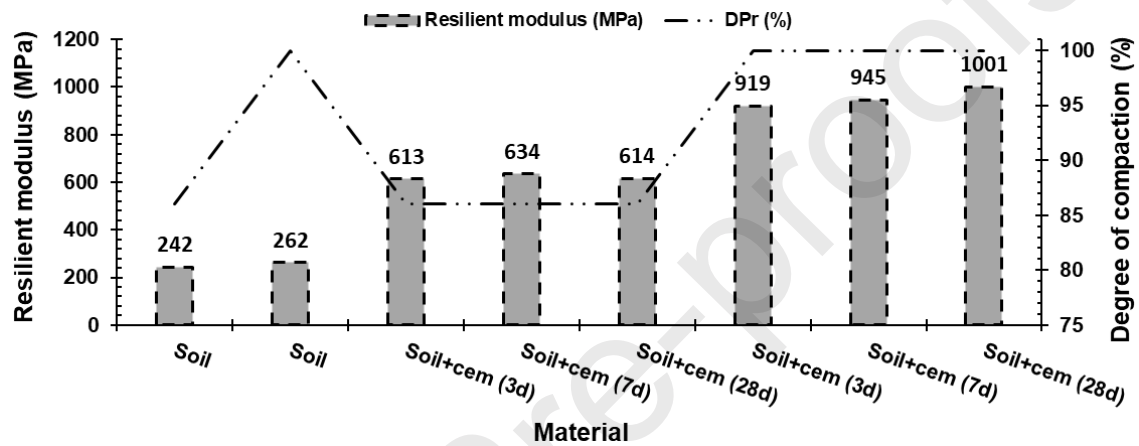


Figure 11. Resilient modulus results for lateritic soil and cement-treated lateritic soil materials

Based on the estimated resilient modulus for the soils, values around 200 MPa were observed for the lateritic soil, which are in accordance with typical granular lateritic tropical soils (Nogami and Villibor, 1981, 1991; Bernucci, 1995; Guimarães et al., 2021; Costa et al., 2022). The Brazilian standard SUP-00048 recommends a minimum of CBR > 20% for the heavy haul railway sub-ballast layer (i.e. such as 32.5 t/axle), which may correspond to a resilient modulus higher than 200 MPa, according to the soil in this research, based on the equations of IP-DE-P00/001 and IPR 719 (2022) Brazilian specifications for tropical soils. The Asphalt Institute (1962) also found similar trend for some soils. Thus, it can be assumed that untreated lateritic soil would be suitable for sub-ballast if compacted with the RFRM vibratory plate regarding its short-term behaviour.

It was also observed that the lateritic soil treated with 3% Portland cement exhibited resilient modulus values three times higher than the untreated condition when compacted under energy levels comparable to those of the RFRM using a vibratory plate and achieving a degree of compaction of 86% of Standard Proctor in accordance with Castro et al. (2025). The resilient modulus increased to four times the untreated value when compacted to 100% of standard Proctor energy. According to DNIT 140-ES (2010), the assessment of road sub-base layers using cement-treated soils must ensure a minimum CBR value of 30%, which approximately corresponds to a resilient modulus of 300 MPa, based on the empirical correlation proposed by the Asphalt Institute (1962). This requirement is consistent with international pavement design guidelines, such as AASHTO (1993), for materials of similar stiffness and bearing capacity.

This suggests that, for this type of soil, treatment with 3% cement may be sufficient to achieve satisfactory short-term mechanical and resilient performance of the sub-ballast even when compacted with the RFRM vibratory plate or below standard Proctor compaction. However, this assumption requires further evaluation, considering that heavy-haul railway loads are substantially more severe than those typically applied in road pavements. This cement content has also been proved effective in the studies from Guthrie et al. (2002), Miller et al. (2006), Beja et al. (2020), Valoura et al. (2022), and Peng et al. (2023). These results highlight the high potential of lateritic gravelly sandy soils to be applied for railway track sub-ballast and heavy haul conditions.

Regarding the silty-sand soil that was assumed in this research for the subgrade soil for in the analysis described later, values of 70 MPa were observed, approximately, leading to its classification as a material with characteristics suitable for simulating a subgrade with medium or fair load bearing capacity. The resilient modulus values were calculated assuming a typical deviatoric stress at the top of the sub-ballast ($\sigma_d = 100$ kPa) for a 32.5 t/axle load under an average operational speed of 80 km/h scenario, based on field monitoring conducted by Costa et al. (2022). It was also assumed confining stresses of $\sigma_3 = 40$ and 100 kPa.

3.2 Cyclic strain accumulation of granular sub-ballast material

Based on the results of degree of compaction obtained from the box test in Castro et al. (2025) and resilient modulus tests performed in this study, permanent deformation behaviour were evaluated through cyclic load triaxial tests conducted for the following materials and scenarios: (A) lateritic soil, compacted with Proctor standard energy $D_{Pr} \approx 100\%$; (B) lateritic soil, compacted with an 86% degree of compaction $D_{Pr} \approx 86\%$, corresponding to the compaction achieved in the experimental box tests; and (C) lateritic soil with cement, compacted with an 86% degree of compaction $D_{Pr} \approx 86\%$, corresponding to the energy applied by the compactor plate in the experimental box tests performed by Castro et al. (2025). The identification of shakedown behaviour was based on the criterion proposed by Werkmeister et al. (2001) and Werkmeister (2003) and adopted in the European standard (EN 13286-7, 2004), which evaluates the accumulated permanent axial strain between 3000 and 5000 load cycles. According to this approach, accumulated strains lower than the Plastic Shakedown Limit (PSL = 0.0045%) indicate shakedown, while values between PSL and the Plastic Creep Limit (PCL = 0.04%) correspond to a transition towards plastic creep. Figure 12 reports the accumulated strain results obtained through laboratory cyclic triaxial tests with the number of load cycles (N) and Table 10 presents the accumulated permanent axial strain obtained between 3000 and 5000 cycles for the different stress states and compaction levels investigated.

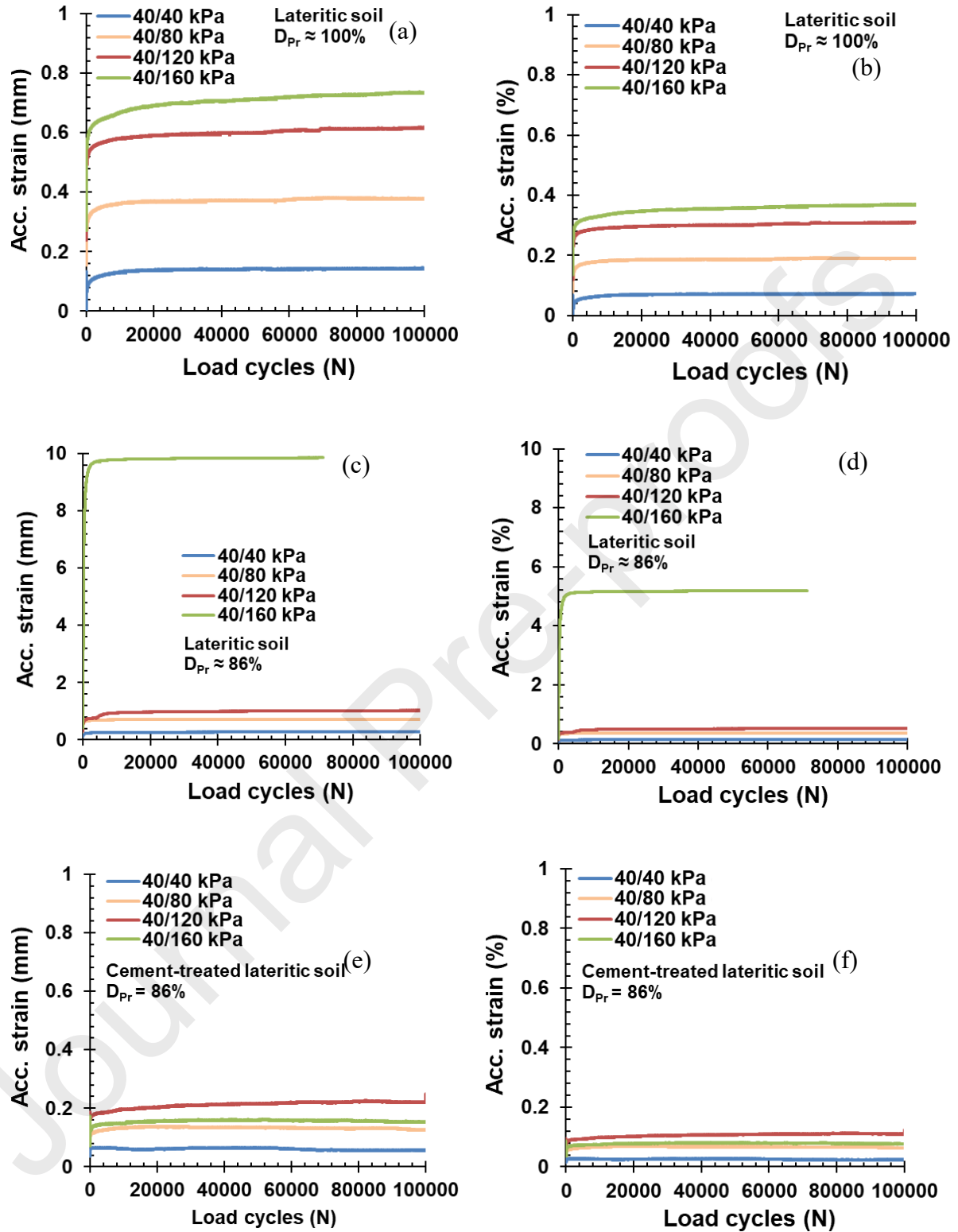


Figure 12. Experimental accumulated strains vs. number of cycles for lateritic soil and cement-treated lateritic soil in mm (in the left) and %-rate (in the right) for different stress states (σ_c/σ_d) and compaction conditions (D_{Pr}): (a) and (b) Standard Proctor energy condition ($D_{Pr} \approx 100\%$); (b) and (c) box test condition ($D_{Pr} \approx 86\%$); (d) (e) box test condition cement-treated soil ($D_{Pr} \approx 86\%$) based on Castro et al. (2024)

Table 10. Accumulated permanent axial strain between 3000 and 5000 load cycles and shakedown classification for evaluated materials

Material / Condition	Degree of compaction	σ_3 (kPa)	σ_d (kPa)	$\Delta\epsilon_p$ (3000-5000 cycles) (%)	Shakedown classification
Lateritic soil	100% Proctor	40	40	0.0025	Shakedown ($\leq$ PSL)
		40	80	0.0044	Shakedown ($\approx$ PSL)
		40	120	0.0058	Transition (PSL-PCL)
		40	160	0.0059	Transition (PSL-PCL)
Lateritic soil	86% Proctor	40	40	0.0027	Shakedown ($\leq$ PSL)
		40	80	0.0046	Transition ($\approx$ PSL)
		40	120	0.0462	Plastic creep ($>$ PCL)
		40	160	0.0351	Plastic creep ($>$ PSL)
Lateritic soil + 3% cement	86% Proctor	40	40	0.0001	Shakedown ($\leq$ PSL)
		40	80	0.002	Shakedown ($\leq$ PSL)
		40	120	0.0014	Shakedown ($\leq$ PSL)
		40	160	0.0011	Shakedown ($\leq$ PSL)

For specimens compacted at 100% of the Standard Proctor energy, the accumulated strains remained close to or below the PSL for all analysed deviatoric stress levels, indicating stable shakedown behaviour despite the gradual increase observed over 100000 cycles which is positive for sub-ballast materials according to DNIT 178 and EN 13286-7 (2004), as shown by the results of scenario A. The favourable physical, chemical and mechanical characteristics of lateritic soils as shown in Guimarães et al. (2021) and Costa et al. (2022) could have contributed to the results. In other words, this response is likely related to its well compaction condition (100% degree of compaction) allied to the inherent structural characteristics of tropical lateritic soils, which commonly present an aggregated and partially cemented fabric

formed by particle clusters bonded by fine lateritic fractions rich in iron and aluminium oxides (Nogami and Villibor, 1995; Bernucci, 1995). Based on these results, this type of material may achieve even higher stiffness values if compacted with the intermediate Proctor energy (i.e., compaction energy between the Standard Proctor and Modified Proctor recommended in Brazilian standards and applied in Brazilian projects) and, consequently, may achieve even lower permanent deformation values due to the higher energy, better interlocking and higher density.

In contrast, for specimens compacted at 86% of the Standard Proctor energy in scenario B, the accumulated strains exceeded the PSL for higher deviatoric stresses, particularly under $\sigma_d = 120$ and 160 kPa, indicating the onset of Plastic Creep. This behaviour is consistent with previous findings reported for railway sub-ballast materials under cyclic loading (Werkmeister et al., 2001; Fortunato et al., 2012). On the other hand, despite it indicated Plastic Creep by the European standard, it was also noticed the stabilisation of deformation around 10 mm for 40 t/axle load ($\sigma_d=160$ kPa) after 5000 cycles, indicating that the permanent strain accumulation tends towards a steady state under the applied cyclic stress. This stress-strain behaviour is consistent with the plastic shakedown limit reported in the literature for Brazilian tropical soils (Guimarães, 2009), where incremental plastic strains diminish with continued loading and the material may reach a shakedown range characterised by a relatively high level of accumulated deformation before treatment (approximately 10 mm) where the material presents significant initial plastic deformation followed accommodation. This highlights the high potential of this type of soil for sub-ballast if well-compacted. Conversely, when cyclic stresses exceed this limit, permanent strain accumulation continues until failure, emphasising the importance of adequate compaction and material selection for long-term track stability.

Accordingly, the results also suggest that when this material or a local poorer soil is compacted using a vibratory plate achieving only 86% of the standard Proctor density and without chemical treatment, it may undergo excessive deformation under heavy-haul traffic conditions, such as 32.5 t/axle or 40 t/axle loading and 120 and 160 kPa vertical stresses, respectively, at the sub-ballast surface. Such behaviour may result in differential settlements, accelerated track geometry degradation, and, in extreme cases, derailments.

In other words, the deformation behaviour of the lateritic sub-ballast soil was strongly dependent on the degree of compaction. The soil exhibited stable long-term behaviour under cyclic loading when compacted at 100% of the Standard Proctor energy, which can be attributed to both its high density and its inherent structural characteristics already reported. In contrast, the soil exhibited increased permanent deformation under 40 t/axle loading when compacted at 86% of the Standard Proctor energy (i.e., representative of the vibratory plate compaction of the RFRM), associated with a higher initial void ratio and reduced stiffness. These results indicate that the observed instability is primarily related to insufficient compaction energy rather than to unfavourable intrinsic properties of the lateritic soil.

Regarding the evaluated cement-treated soil in scenario C, the results showed that the lateritic soil exhibits very low permanent deformation values and shakedown behaviour, predominantly, even when compacted at 86% and submitted to a 40 t/axle load operational condition, based on EN 13286-7 (2004) and Guimarães et al. (2021), ensuring that the structural and operational quality of the railway track is maintained. It should be noted that the shakedown criteria adopted in this study, originally proposed by Werkmeister (2003) and incorporated in European standards for unbound granular materials, were not specifically developed for cement-treated soils. Therefore, their application to the stabilized lateritic material should be interpreted in a qualitative and comparative sense rather than as a strict acceptance criterion. Nevertheless, the markedly lower accumulation of permanent strain observed for the soil treated

with 3% cement, even under higher deviatoric stress levels, provides strong evidence of a stable long-term response and supports the effectiveness of cement stabilization under cyclic loading.

Thus, to mitigate this deficiency in compaction using the vibratory plate and ensure strain accumulation stabilisation over a high number of cycles, the importance of chemically treating the material with chemical binders, such as cement, becomes evident, mainly when lateritic soils are not available in the railway track region. The improved behaviour of the cement-treated lateritic soil may be associated with cement hydration and pozzolanic reactions between calcium hydroxide and the silica- and alumina-rich clay fractions typical of lateritic soils. These reactions can lead to the formation of different cementitious compounds such as calcium silicate hydrates (C-S-H), calcium aluminate hydrates (C-A-H), and calcium aluminosilicate hydrates (C-A-S-H) which can enhance interparticle bonding and stiffness and reduce permanent deformation under cyclic loading (Amadi et al., 2025).

Nevertheless, the improved permanent deformation response of the 3% cement-treated material should not be interpreted as the absence of all long-term deterioration mechanisms. Cement addition leads to a more bonded or semi-rigid response, which is favourable for controlling rutting and settlement but may also make shrinkage and fatigue cracking relevant under prolonged service loading (Ceratti, 1991; Duan et al., 2023). These mechanisms are not captured by the present HCA formulation and therefore require a dedicated bonded-material or damage-based assessment before a definitive design recommendation is established.

In general, lateritic soil tended to achieve shakedown and transition regime for most of the stress state conditions for degree of compaction of 86 and 100% without cement treatment, except for the scenario which would represent 32.5 and 40 t/axle load and $D_{Pr} \approx 86\%$ (plastic creep regime). This evidences that the investigated RFRM vibratory compactor plate is not a proper method for sub-ballast compaction to satisfy this load increase.

To conclude, the addition of 3% cement is a technically effective solution when vibratory plate compaction is adopted, especially in situations where untreated lateritic soil at low compaction would accumulate excessive permanent deformation. However, a complete design recommendation for cement-treated sub-ballast should also consider the potential for cracking and stiffness degradation associated with its more semi-rigid response. In contrast, if the material is not chemically treated, it should be evaluated whether an increase in compaction energy from standard to modified Proctor would be sufficient to meet this specific design scenario of 40 t/axle. Even though the use of cement is not commonly sustainable, it may become more sustainable when there are no available materials near the construction site, which would otherwise demand higher carbon emissions from transporting materials over long distances. Additionally, cement may be replaced with alternative binders such as lime, bio-binders, bio-asphalts, or binders from reclaimed asphalt mixtures, considering their lower CO₂ emissions during production and reuse, and the acceptable performance they provide when applied in pavements and railways.

3.3 HCA parameters calibration

The cyclic triaxial tests were also performed to calibrate the HCA parameters to be applied in the long-term behaviour analysis of the railway track sub-ballast subjected to train loads (black continuous line with circles in Figure 13). In other words, the parameters were

obtained and compared with the results of the railway track model (dotted red line). Thus, Figure 13 presents the accumulated permanent strain as a function of the number of load cycles obtained from cyclic triaxial tests, together with the corresponding fitting curves $\dot{\epsilon}_N$ based on the $\epsilon^{acc}(N)$ results to obtain the parameters C_{N1} , C_{N2} and C_{N3} used for HCA model calibration.

The cyclic load triaxial tests results demonstrated to adjust well with the logarithmic $\dot{\epsilon}_N$ function applied to calculate the accumulated strain as a function of the number of cycles and recommended by Wichtmann (2005) and Niemunis et al. (2005) to determine the C_{N1} , C_{N2} and C_{N3} parameters for the different analysed stress states and two initial void ratios. However, according to Wichtmann et al. (2015), a deeper evaluation related to the variation of the strain accumulation with three different initial void ratios could be more accurate, due to the C_e parameter determination from laboratory cyclic triaxial tests. The hypoplastic and HCA model parameters applied are shown in Table 11 and Table 12.

Table 13 shows a comparison between the parameters obtained for the well-graded granular material in this research and in Wichtmann et al. (2010) determined for a typical UGM pavement base layer in the literature.

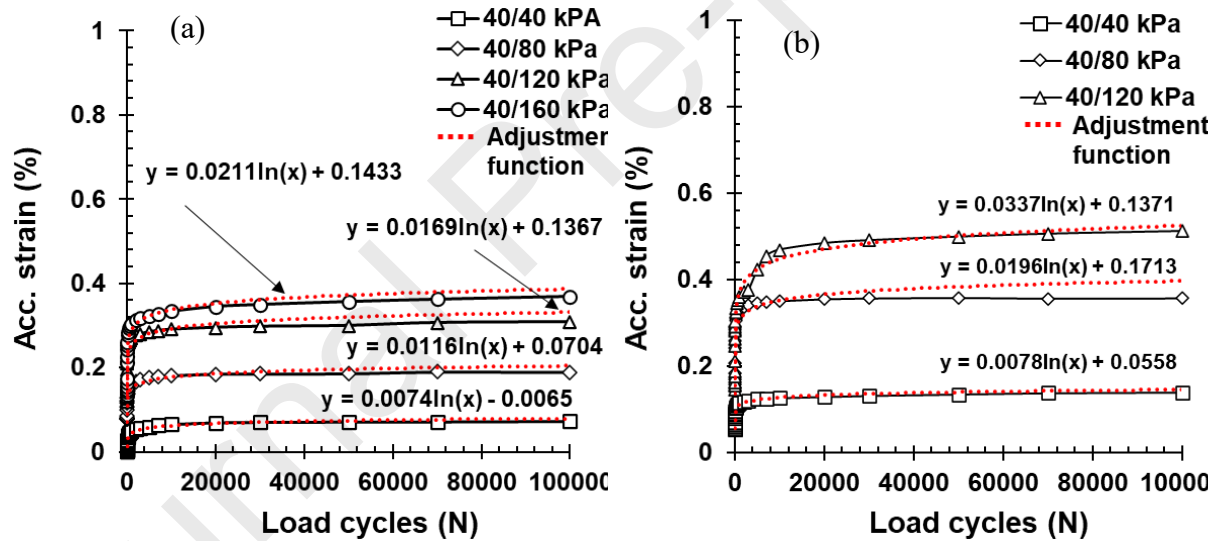


Figure 13. Accumulated strains vs. number of cycles fitting curves for the studied compacted lateritic soil, based on the experimental data: (a) Standard Proctor compaction condition ($D_{Pr} \approx 100\%$) cyclic triaxial test results; (b) box test compaction condition ($D_{Pr} \approx 86\%$) according to Castro et al. (2024) for a RFRM plate compaction

Table 11. Key hypoplastic model parameters

Variable UMAT (ABAQUS/CAE)	Stress state condition - kPa			
	40 to 40	40 to 80	40 to 120	40 to 160
Hypoplastic model parameters - implicit mode				
Critical friction angle - φ_c (rad)	0.63	0.63	0.63	0.63
Poisson's ratio - ν	0.32	0.32	0.32	0.32
Granular hardness - h_s	300000	300000	4000000	2550000
Bauer's compression rule exponent - n	0.25	0.36	0.28	0.20
Minimum void ratio - e_{d0}	0.46	0.46	0.46	0.46
Critical void ratio - e_{c0}	1.231	1.231	1.231	1.231
Maximum void ratio - e_{i0}	1.408	1.408	1.408	1.408
Constant α	0.12	0.12	0.12	0.12
Constant β	1.9	1.9	1.9	1.9
Stiffness multiplier - m_T	1.1	1.1	1.1	1.1
Stiffness multiplier - m_R	2.2	2.2	2.2	2.2
$R_{\max}$	0.0001	0.0001	0.0001	0.0001
β_R	0.2	0.2	0.2	0.2
χ	6	6	6	6

Table 12. Key HCA model parameters

HCA model parameters - explicit mode				
Variable UMAT (ABAQUS/CAE)	Stress state test condition - kPa			
	40 to 40	40 to 40	40 to 40	40 to 40
C_{N1}	0.0074	0.0120	0.0169	0.0211
C_{N2}	1.40	1.80	2.46	1.50
C_{N3}	0	0	0	0
C_p	-0.06	-0.06	-0.06	-0.06
C_{p1}	1.7	1.7	1.7	1.7
C_{p2}	0	0	0	0
C_Y	3.2	3.2	3.2	3.2
C_{e1}	0.41	0.41	0.41	0.41
C_{e2}	0	0	0	0
$C_{\pi1}$	1	1	1	1
$C_{\pi2}$	1	1	1	1
Atmospheric (reference) pressure - p_{atm}	100	100	100	100
Reference void ratio - e_{ref}	1.231	1.231	1.231	1.231
Reference amplitude - ε_{ref}	0.0001	0.0001	0.0001	0.0001
Material constant for elastic stiffness - A_K	1300			
Material constant for elastic stiffness - a_K	0.46			

Material constant for elastic stiffness - n_K	0.32
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Table 13. Comparison between lateritic soil and UGM material parameters

HCA model parameters			
Description	Variable	Method (b) calibration – (10^5) cycles	Wichtmann et al. (2010)
Diameter passing 50%	d_{50}	4.0	6.3
Material coefficient	C_y	3.2	1.8
Material coefficient	C_e	0.44	0.07
Material coefficient	C_p	-0.06	-0.22
Material parameter	C_{amp}	1.70	1.10
Minimum void ratio	e_{ref}	1.224	0.230
Hypoplastic model parameters			
Maximum void ratio	e_{max}	1.408	0.440
Minimum void ratio	e_{min}	0.460	0.230
Critical void ratio	e_{c0}	1.231	0.383
Critical friction angle	φ_c	36	38

To verify the validity and accuracy of the obtained parameters, it was developed some finite element axis-symmetric analysis of cyclic triaxial tests with the same stress state and initial density conditions than those performed in laboratory. Figure 14 shows the results for the finite element analysis with the tested and calibrated models' parameters.

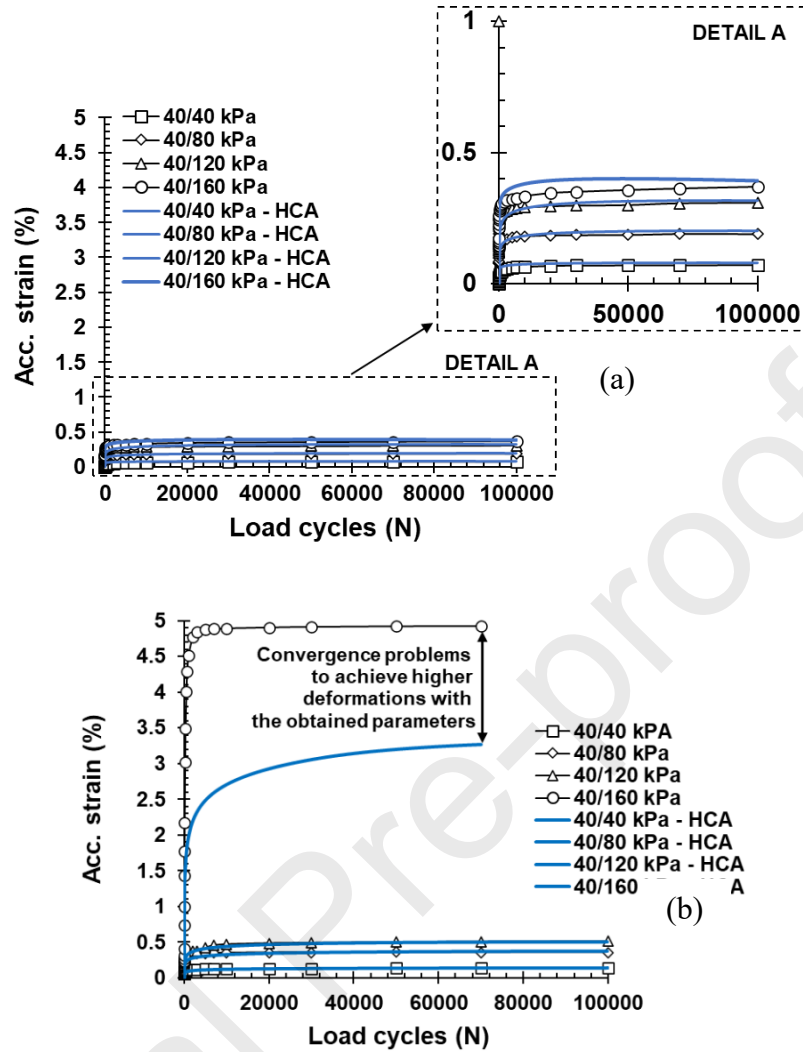


Figure 14. Laboratory vs. finite element analysis accumulated strain results: (a) Standard Proctor condition ($D_{Pr} \approx 100\%$); (b) (c) box test condition ($D_{Pr} \approx 86\%$)

According to Figure 14, most of the simulated stress states reproduced the laboratory trends satisfactorily, except for the 40/160 kPa condition, for which numerical convergence was not achieved at higher deformation levels. This behaviour is interpreted as an indication that the response is leaving the reliable domain of the present calibration and modelling assumptions rather than as a physically converged prediction. The difficulty is consistent with the fact that the HCA model was originally developed for very small strain amplitudes ($\varepsilon^{amp} \leq 10^{-3}$) and medium-coarse sandy materials, whereas the lateritic soil investigated here exhibited strain amplitudes on the order of 10^{-2} and contains silt and clay fractions (i.e., cohesive fraction). In addition, the absence of monotonic triaxial tests for some hypoplastic parameters and the uniaxial nature of the cyclic triaxial measurements may also have influenced the accuracy of the implicit-explicit coupling used in the simulations. Because the laboratory program did not quantify strain-loop polarization or the rotation of principal strain axes ($f_{\pi}=1$), the calibration cannot fully represent the multiaxial loading features known to affect accumulation rates in the original HCA formulation (Niemunis et al., 2005). Figure 15 shows the experimental strain amplitude evolution with load cycles.

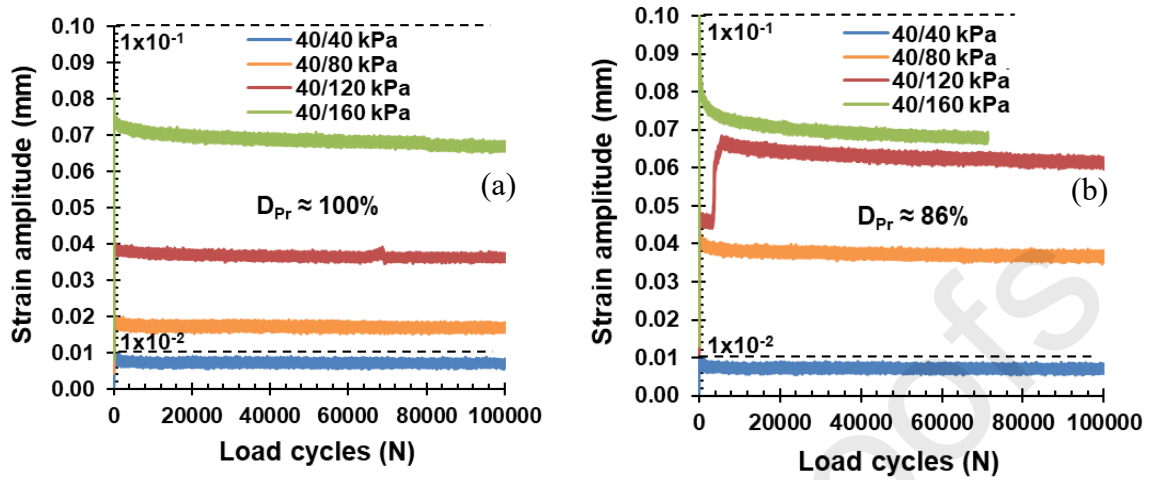


Figure 15. Experimental strain amplitudes vs. load cycles for lateritic soil in mm: (a) Standard Proctor energy ($D_{Pr} \approx 100\%$); (b) box test compaction condition ($D_{Pr} \approx 86\%$)

The applicability of the HCA model must therefore be interpreted in light of both strain-amplitude dependency and material type. According to Niemunis et al. (2005) and Wichtmann (2016), the amplitude-dependent function f_{amp} governing the accumulation rate is validated for strain amplitudes up to $\varepsilon^{amp} \approx 10^{-3}$. For larger strain amplitudes, experimental evidence indicates that the accumulation rate tends to become nearly independent of strain amplitude, a behaviour accounted for in the model by limiting f_{amp} to a constant value. Consequently, although cyclic triaxial tests on the investigated lateritic soil involved strain levels exceeding 10^{-3} , the HCA formulation can only be interpreted here as a phenomenological approximation for long-term accumulation trends and not as a detailed constitutive description of large-strain behaviour.

This interpretation is consistent with the simplified calibration procedure recommended by Wichtmann (2016) as a minimum standard, in which parameters are derived from a limited number of cyclic triaxial tests without requiring an explicit multi-amplitude calibration. Nevertheless, the present lateritic material contains cohesive fractions and an aggregated fabric that differ from the sands for which the model was originally formulated. Therefore, the HCA framework is adopted herein only for the untreated lateritic sub-ballast within stable to moderately accumulative regimes, where it is used to compare long-term trends between compaction scenarios. No ad hoc modification of the strain-amplitude function was introduced for the bonded or cement-treated material because such a reformulation would not be sufficiently supported by validated calibration data.

In general, the converged results were consistent, as accumulation increased with higher deviatoric stress while strain amplitude and confining stress were kept approximately constant, as expected (Wichtmann et al., 2010; 2015). Some parameters for the studied untreated lateritic soil could therefore be calibrated for HCA application. However, the numerical predictions should be regarded as representative only within the converged stress states and within the applicability boundaries discussed above. The cement-treated lateritic soil was not evaluated through the HCA model because the present formulation was not developed for soil-cement

mixtures or other chemically bonded materials. Its long-term behaviour was therefore assessed experimentally through cyclic triaxial tests. Supported by these experimental results (Castro et al., 2025) and by the untreated-soil numerical trends, cement stabilization with 3% was identified as an effective measure to control permanent deformation when vibratory plate compaction is used.

3.4 Nonlinear long-term behaviour of the railway track sub-ballast

3.4.1 UMAT sub-routine verification, meshing, load frequency - railway track modelling and structural analysis

Figure 16 illustrates the results referred to an axis-symmetric analysis of a cyclic triaxial test with a 0.10×0.10 m sample.

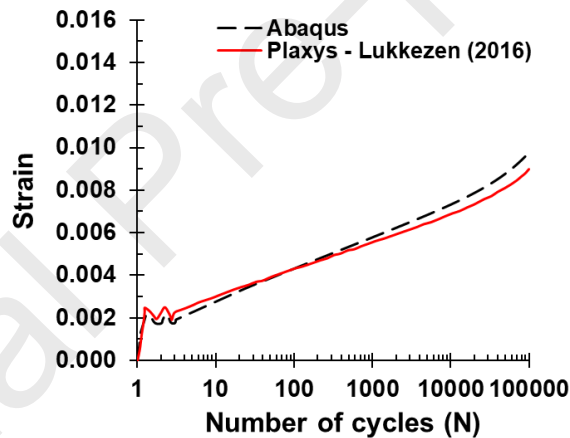


Figure 16. Axis-symmetric strain accumulation results ($e_0=0.828$)

It could be demonstrated satisfactory results for accumulated permanent deformation for 1×10^5 cycles, when compared to those obtained by Lukkezen (2016).

The modelling of the nonlinear long-term behaviour of the railway track initiated by performing a meshing configuration and load frequency analysis to show their influence on the strain and displacement results of the railway track subjected to cyclic loads from the traffic. Figure 17 depicts the strain accumulation in the sub-ballast as a function of the number of cycles for each mesh configuration assessed within the developed three-dimensional railway model, considering the weighted average of the centroid strains. Table 14 presents the results of the mesh discretisation analysis, specifically detailing both the computational time and the number of elements for each mesh examined.

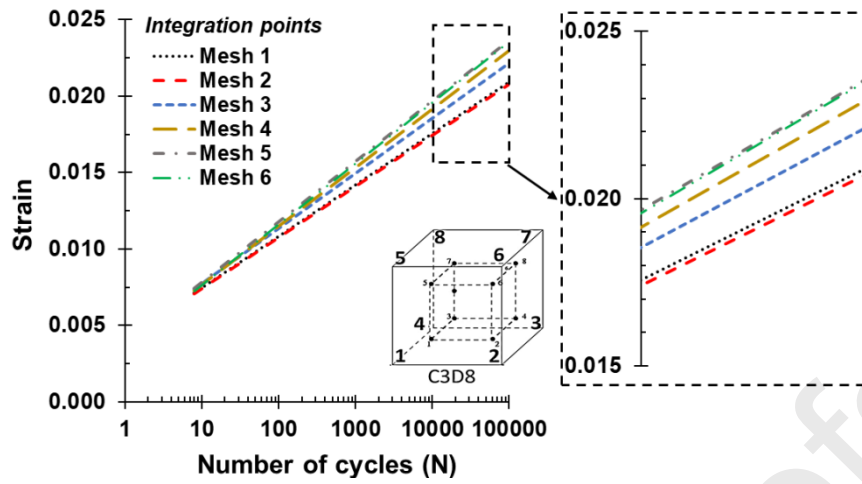


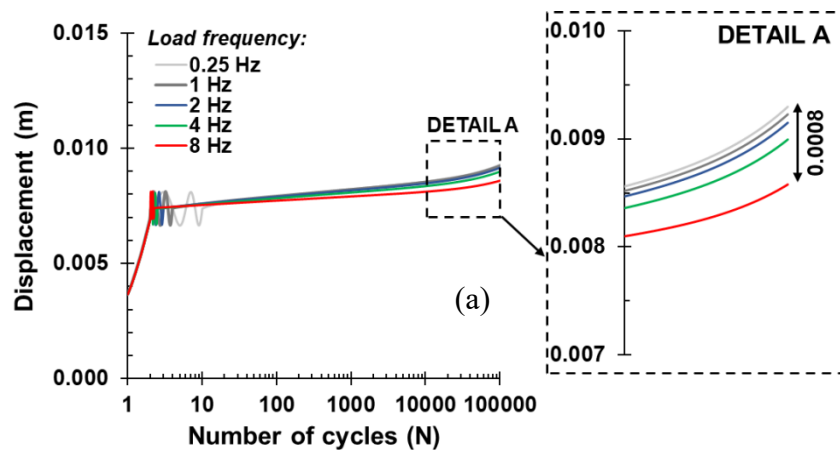
Figure 17. Strain accumulation in the sub-ballast as a function of the number of cycles for each mesh configuration

Table 14. Mesh discretization analysis

Test	Elements No.	Analysis (time)	Increase in analysis time	Increase in elements
		h:m:s		
Mesh 1	1280	00:09:00	-	-
Mesh 2	2096	00:12:00	$\times 1.3$	$\times 1.6$
Mesh 3	3974	00:21:00	$\times 2.3$	$\times 3.1$
Mesh 4	8832	00:44:00	$\times 4.9$	$\times 6.9$
Mesh 5	15700	01:09:00	$\times 7.7$	$\times 12.3$
Mesh 6	52696	06:00:00	$\times 40.0$	$\times 41.2$

h - hour, m - minute, s - second

According to the performed analyses, it was observed that a reasonable cost-benefit balance was achieved with mesh 5. It was noted that increasing the number of elements by 41 times, as seen with mesh 6, resulted in a significant computational time increase of 40 times. This solution is not considered efficient from the perspective of the results obtained, given that mesh 5 provided similar strain accumulation results with only a 7-fold increase in computational time. Figure 18 and Table 15 present the results of the influence of the load application frequency in the 3D railway simulation using the HCA model, which considered the range from 1 to 8 Hz by sinusoidal periodic loads.



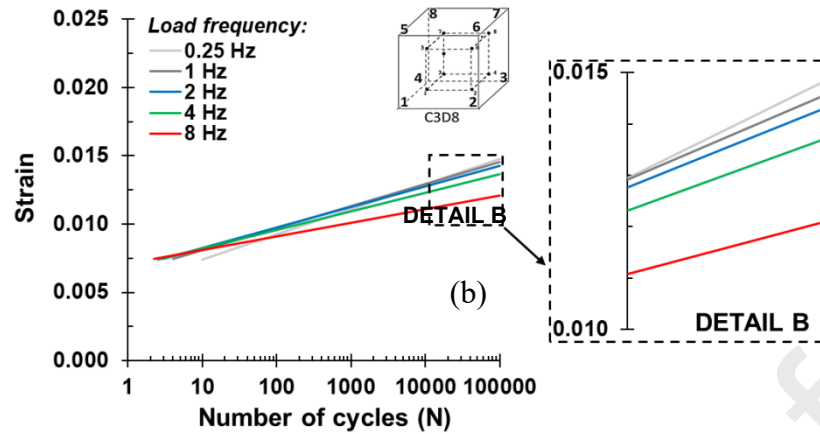


Figure 18. Load frequency analysis results: (a) displacement at the top of the sub-ballast layer; (b) strain of the sub-ballast layer

Table 15. Frequency analyses

Analysis	Steps (s)				Frequency (Hz)	Displacement (m)	Strain (%)
	1	2	3	4			
1	1	1	4	4	0.25	0.0093	1.479
2	1	1	1	1	1	0.0092	1.45
3	1	1	0.50	0.50	2	0.0092	1.427
4	1	1	0.25	0.25	4	0.0090	1.367
5	1	1	0.13	0.13	8	0.0086	1.208
Difference between analysis 1 and 5						0.0008	0.271%
Relative Difference between analysis 1 and 5						8.6%	18.3%

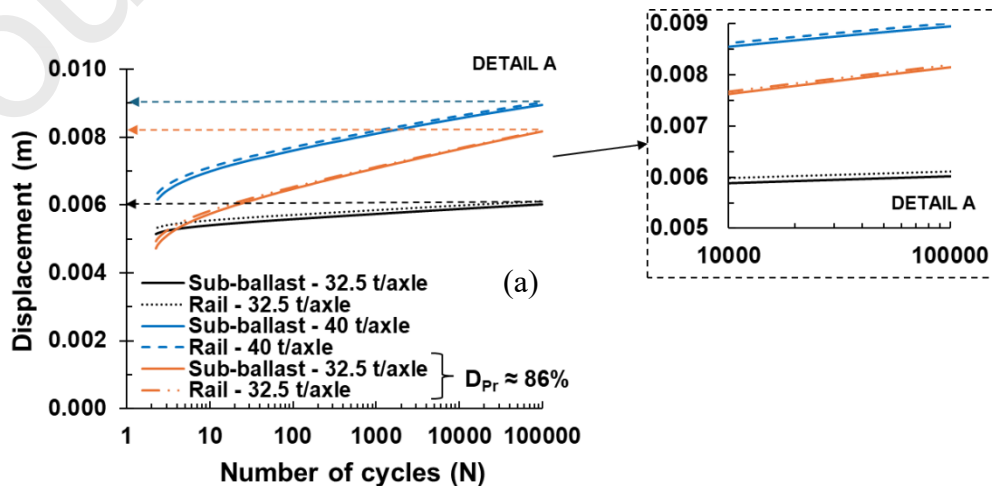
The analyses also showed that the difference in deformation when using a frequency of 1 Hz or 8 Hz is very small, as reported by Witchmann (2005). Thus, it is understood that for this type of analysis, a frequency of 8 Hz can be considered, which was assumed closer to the actual field load application for freight train compositions traveling at 80 km/h and 2.40 m axle

spacing. Figure 19 exhibits the obtained results of the nonlinear elastic long-term behaviour analysis of the railway track focusing on the displacement and the vertical stresses in the top of the sub-ballast layer and the strain accumulation and the void ratio variation of the analysed region.

Based on the results, it was observed that the sub-ballast layer composed of the evaluated lateritic soil material, exhibited lower accumulated deformations if compacted at 100% of the standard Proctor energy, tending toward shakedown, as previously demonstrated by experimental and modelling results for the current operational condition of 32.5 t/axle. However, it was also noted that deformations still showed an increasing trend for the 1×10^5 loading cycles analysed. These findings confirm that this type of material would not exhibit satisfactory mechanical behaviour under the current scenario of 32.5 t/axle, if compacted using the RFMR compaction plate or with a degree of compaction below 100% of the standard Proctor.

The difference in the shape of the displacement and accumulated strain curves is due to the mode and frequency of recording of both output results. The displacement results, calculated at a node located at the top of the sub-ballast, were recorded using the history output mode, which stores data at every increment for selected variables, thereby providing more precise and frequent information. This higher accuracy, however, entails greater computational cost and larger data files. In contrast, the accumulated strain results, calculated in the elements constituting the sub-ballast layer, were recorded using the field output mode, which stores data at selected intervals and is less detailed, making it faster and less demanding in terms of computation. Consequently, the choice between these two methods affected both the frequency of the recorded results and the overall efficiency of the analysis.

This assertion can be substantiated by observing the trend of accumulated deformations in the scenario with a degree of compaction of 86%, verified through test box experiments. In other words, although the studied lateritic soil is considered a high-quality material for pavement construction due to its grain size distribution, chemical and mineralogical characteristics (Nogami and Villibor, 1991; Bernucci, 1995), its compaction using the FRM compaction plate is not recommended for an increase in design axle load to 40 t/axle, even when compacted at 100% of the standard Proctor, as illustrated by the Figure 19.



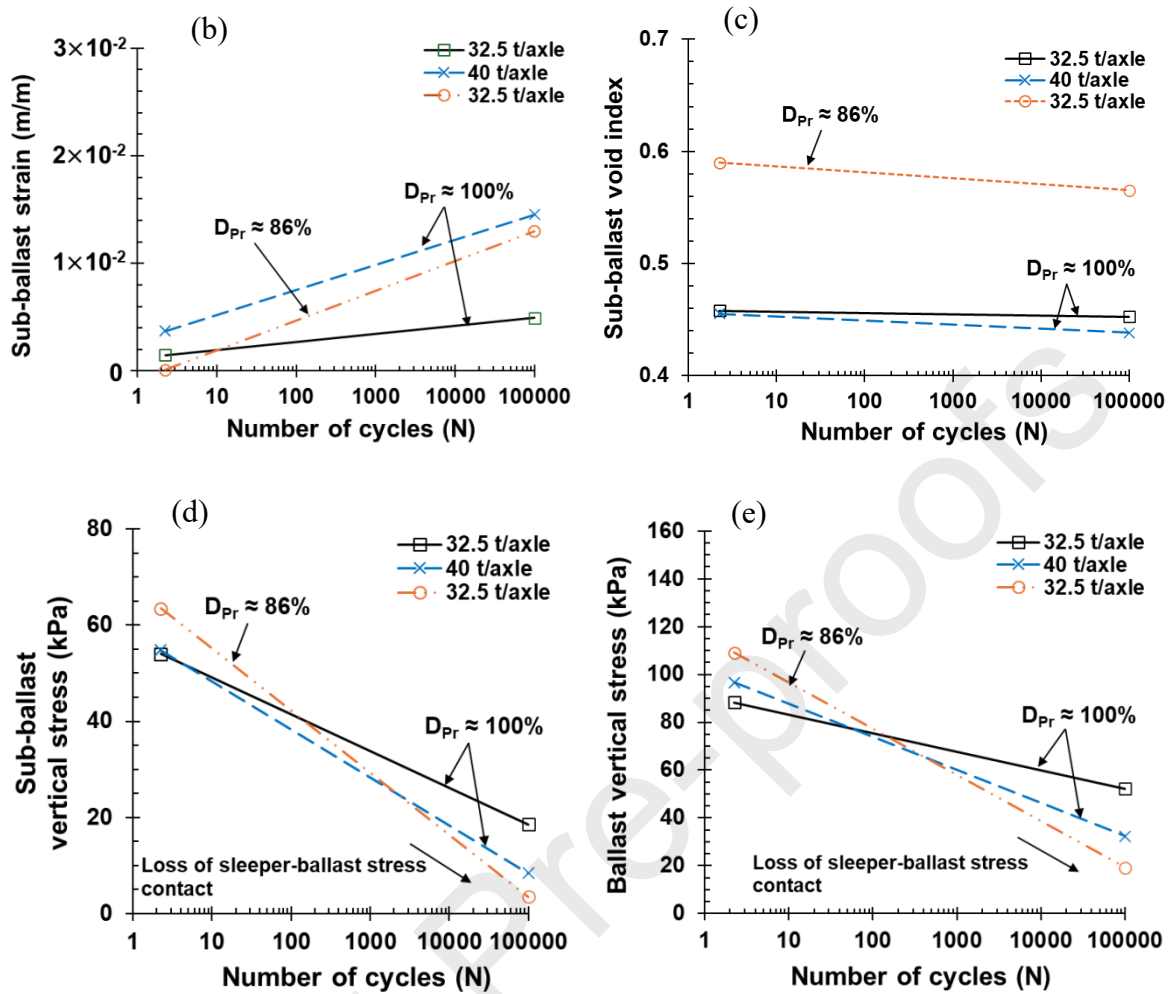


Figure 19. (a) Vertical displacement results vs. number of cycles at the top of the sub-ballast layer; (b) Accumulated strain results of the sub-ballast layer; (c) Void ratio variation of the sub-ballast layer; (d) Vertical stresses at the sub-ballast layer; (e) Vertical stresses at the ballast layer vs. Number of cycles

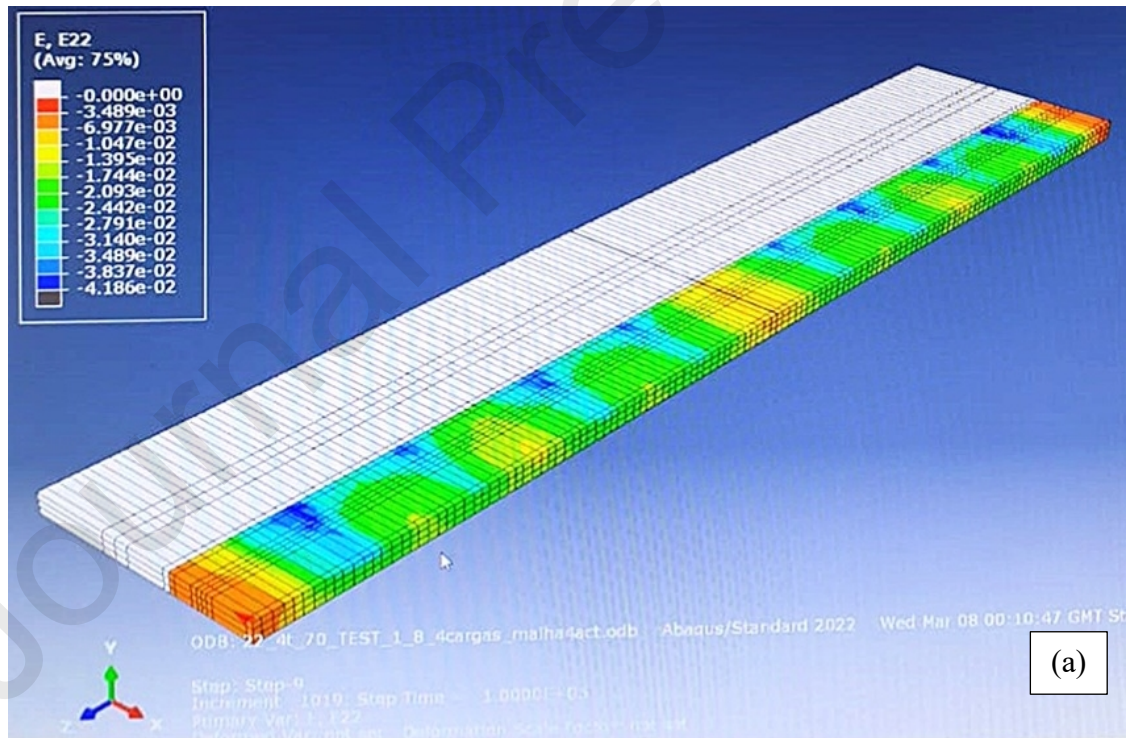
Considering the results of void ratio variation, the same behavioural trend was observed. The materials compacted at 100% of the standard Proctor energy started with the same void ratio, showing greater variation under the more critical condition of 40 t/axle compared to the current scenario of 32.5 t/axle. Meanwhile, the material compacted at 86% of the degree of compaction initially had a higher void ratio, exhibiting a greater decrease due to its looser structural condition and higher porosity.

Finally, the vertical stress results showed initial values lower than those found in the linear elastic modelling of the railway track reported by Castro et al. (2025), followed by a gradual decrease over the loading cycles. Part of this behaviour is associated with the progressive accumulation of permanent deformation in the sub-ballast and the corresponding redistribution of contact pressures at the ballast-sub-ballast interface. However, the interpretation of the absolute stress reduction must be made with caution because the ballast layer was represented as linear elastic. This means that ballast particle rearrangement, degradation, and plastic accommodation were not explicitly captured, although these

mechanisms are known to influence contact conditions and stress redistribution in real ballasted tracks.

Since ballast and sub-ballast were modelled as separate layers interacting through friction, and the load was applied cyclically at a fixed location rather than moving along the track, the computed reduction in contact and vertical stresses reflects the combined effect of repeated loading at the same point and the constitutive asymmetry between a linear elastic ballast layer and a nonlinear HCA-based sub-ballast layer. In this framework, part of the apparent loss of contact at the ballast-sub-ballast interface may be exaggerated because the ballast cannot plastically readjust or rearrange to accommodate the accumulated deformation of the sub-ballast. Therefore, the exact magnitude of interface stress reduction should not be overinterpreted as a fully realistic prediction. Nevertheless, because the same ballast modelling assumption was used consistently in all simulations, the comparative trends observed between compaction conditions remain informative, particularly the greater deformation susceptibility of the untreated material at lower compaction.

This phenomenon can be better shown through the longitudinal deformation results of the sub-ballast layer, after conducting the final analysis with a sandy soil from Wichtmann (2005) applying the developed model as shown in Figure 20 and Figure 21.



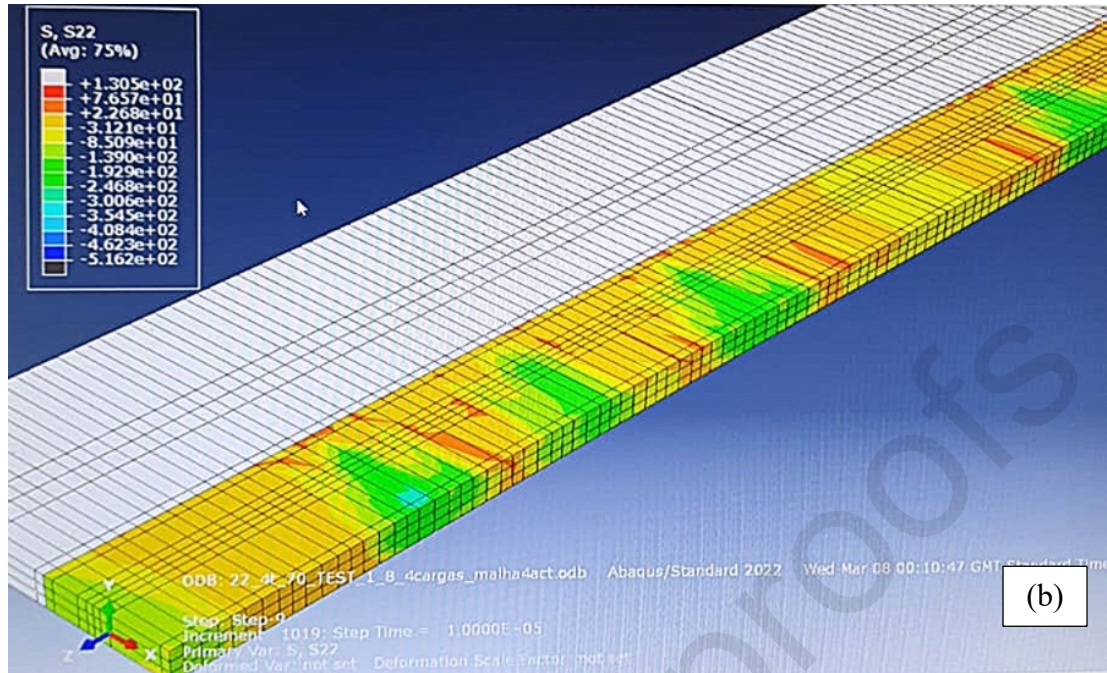
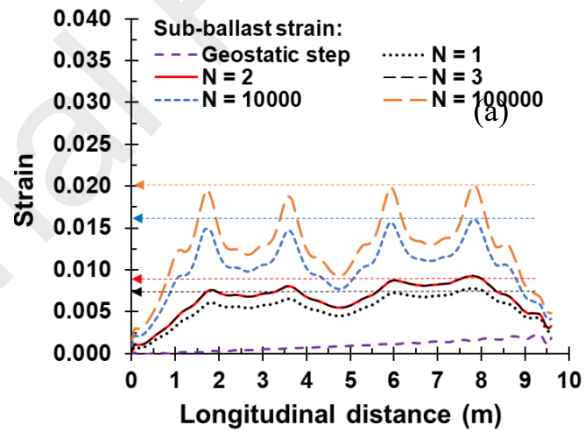


Figure 20. Example of some results obtained through the HCA model computed at the sub-ballast layer: (a) accumulated strain E_{22} [in m]; (b) vertical stresses S_{22} [in kPa]



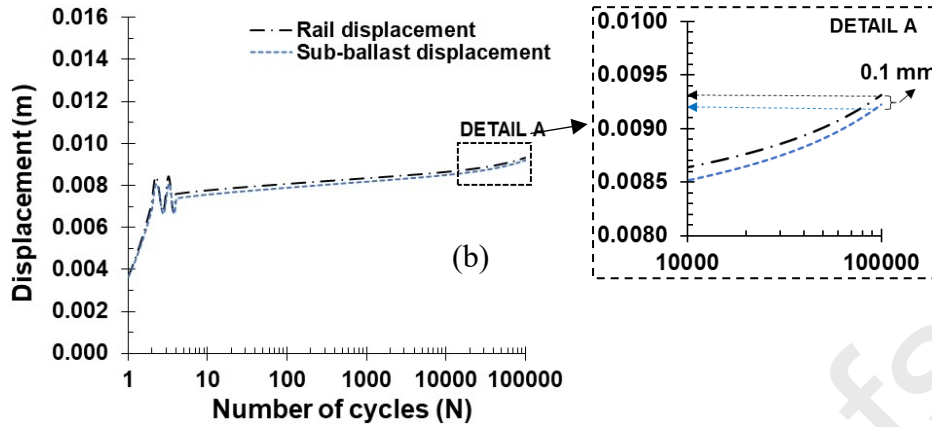


Figure 21. (a) Vertical strain accumulation vs. longitudinal distance at the sub-ballast layer over the number of cycles; (b) Vertical displacement vs. number of cycles at the rail bottom and sub-ballast top

Although the adoption of a nonlinear ballast model could influence the absolute magnitude of stresses and contact conditions, the relative trends observed in sub-ballast deformation and the comparative evaluation of material performance are not expected to change. These limitations are acknowledged, and the incorporation of nonlinear ballast behaviour is recommended for future studies aiming at more detailed stress-deformation analyses.

The numerical simulations performed in this study adopt an equivalent cyclic loading scheme applied at a fixed location rather than explicitly modelling moving train loads and full dynamic interaction. This simplification was adopted to enable a computationally efficient assessment of long-term deformation behaviour over a very high number of load cycles, which would be impractical using fully dynamic moving-load analyses within the scope of the present study. Nevertheless, the assumption neglects stress path rotation, load migration along the track, and transient dynamic effects associated with wheel passage, and therefore it is expected to underestimate accumulated permanent deformation, especially under the more severe 40 t/axle condition.

In particular, previous studies have shown that stress rotation and non-proportional loading can accelerate strain accumulation in granular materials (Niemunis et al., 2005; Varandas et al., 2020; Paixão et al., 2021). Because the present laboratory and numerical framework do not explicitly reproduce PSAR, it is not possible to define a robust quantitative error range for this simplification without additional dedicated simulations. Consequently, the results presented herein should be interpreted as a simplified, comparative, and conservative representation of long-term behaviour, suitable for evaluating relative performance between compaction methods and material treatments rather than for reproducing the full dynamic response of the track. Future work should incorporate three-dimensional dynamic analyses with moving loads to better quantify stress path evolution, load redistribution, and their influence on long-term deformation under heavy-haul railway traffic.

4 CONCLUSIONS

This study investigated the long-term deformation behaviour of tropical lateritic soils used as railway sub-ballast, considering laboratory assessment of both untreated and cement-treated conditions under heavy-haul loading and three-dimensional numerical modelling of the untreated material within the High-Cycle Accumulation (HCA) framework. Based on these results, the following main conclusions can be drawn:

- The investigated lateritic soil exhibited stable long-term behaviour when compacted at 100% of the Standard Proctor energy under the current and more demanding operational loading of 32.5 and 40 t/axle, respectively, reflecting the combined effect of high density and its inherent aggregated and partially cemented structure, which provides adequate stiffness and resistance to cyclic loading.
- The soil showed increased permanent deformation, particularly under 40 t/axle loading, when the soil was compacted at 86% of the Standard Proctor energy, representative of the vibratory plate compaction of the Rail-mounted Formation Rehabilitation Machine (RFRM), indicating that this compaction method alone is inadequate for heavy-haul upgrade scenarios.
- Cement treatment with 3% proved effective in significantly reducing permanent deformation and ensuring stable long-term behaviour in the laboratory tests under both axle load levels, compensating for limitations associated with insufficient compaction energy of the RFRM vibratory plate. These findings demonstrate that adequate compaction and/or chemical stabilization are essential to ensure the long-term stability of lateritic sub-ballast layers in heavy-haul railways. However, because cement treatment changes the material response toward a more bonded or semi-rigid behaviour, additional mechanisms such as fatigue or shrinkage cracking should also be evaluated before establishing a definitive design recommendation for cement-treated sub-ballast.
- The developed HCA-based three-dimensional numerical model showed good agreement with laboratory results for the untreated lateritic soil within the converged stress states and proved to be a useful tool for assessing long-term strain accumulation and comparing compaction scenarios. Its application should nevertheless be restricted to untreated lateritic material and interpreted within the strain-amplitude and material-type limitations discussed in this paper.
- Overall, the results indicate that tropical lateritic soils can be suitable sub-ballast materials for heavy-haul railways, provided that adequate compaction and/or chemical stabilization is applied. The study also highlights the limitations of vibratory plate compaction for increased axle loads and shows that the fixed-point HCA-based simulations should be interpreted as comparative and conservative with respect to accumulated deformation because PSAR and moving-load effects were not explicitly modelled.
- Although cement stabilization provides well-established mechanical performance under heavy cyclic loading, alternative binders such as lime, bio-binders, and reclaimed asphalt materials offer potential environmental benefits. However, their long-term durability and deformation behaviour under heavy-haul railway conditions remain less documented than those of cement-treated soils, indicating the need for further investigation before widespread application.
- From a practical perspective, the results highlight the importance of compaction quality and demonstrate that numerical modelling using the HCA framework can support more reliable design and rehabilitation of railway substructures in tropical regions. For cement-treated

solutions, however, settlement control should be complemented in future work by dedicated fatigue and damage assessment.

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