



Fracture energy regularization for the numerical modelling of concrete structures with recycled aggregate: An application with an UMAT user subroutine for ABAQUS

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ABSTRACT

This paper investigates the application of a new concrete damage model for the numerical simulation of recycled aggregate concrete (RAC) structures. The proposed model incorporates fracture energy regularization for tensile and compressive behaviour in Mode I, addressing mesh dependency issues common in finite element (FE) analyses. Using the ABAQUS FE software, the damage model was implemented via a UMAT user subroutine developed in Fortran 90. The accuracy of the numerical model was evaluated by comparing its results with experimental data from previous studies, focusing on structural responses, cracking patterns, and the estimation of tensile and compressive fracture energy for concrete with recycled aggregates in structural members. The findings from the simulation of two classical concrete benchmark tests, conducted on concretes with different percentages of recycled aggregates, validate the adopted material properties for future applications.

1. Introduction

Concrete is the most widely used material in construction and has experienced increased demand due to rapid urbanization and advancements in infrastructure development. Nevertheless, this increased usage comes with significant challenges, such as the extensive depletion of non-renewable resources and the generation of substantial construction waste, which contradict global efforts toward environmental conservation and sustainable development [1]. Therefore, recycled aggregate concrete (RAC) and its components have gained significant attention from researchers [2–4].

Currently, the replacement of aging buildings and infrastructure generates large quantities of construction debris, especially concrete waste. Since aggregates constitute the majority of concrete's volume, recycling these materials from demolition waste to produce new concrete is a logical step [5,6]. Many researchers have concentrated on the utilization of recycled coarse aggregates (RCAs) [7–9], and both RCAs and concrete waste processing techniques have gained popularity due to their economic and environmental benefits [10,11]. The use of RCAs reduces landfill waste and the demand for natural coarse aggregates (NCAs), thereby mitigating environmental pollution. However, despite their demand, RCAs are primarily used in road bases and non-structural

concrete, as their quality is less consistent compared to NCAs, limiting their use in structural applications.

The construction industry's environmental impact is further exacerbated by its heavy reliance on natural, non-renewable resources and the significant volumes of construction and demolition waste (CDW) it generates. Recycling CDW offers an effective means of reducing the ecological footprint of construction, particularly by replacing natural aggregates in concrete production. While many countries have started to accept coarse recycled aggregates for certain practical applications, the use of fine recycled aggregates remains restricted or prohibited. This restriction limits the utilization of a substantial portion of CDW, up to 50%, due to issues such as high-water absorption and the presence of contaminants.

1.1. Objectives

The main objective is to test a new concrete damage model using the commercial finite element (FE) software ABAQUS, to simulate the structural behaviour of RC beams with RAC. The numerical results are compared with experimental data provided by the scientific community. The concrete constitutive relation was implemented using a user-subroutine UMAT with Fortran 90 code. This model uses the concrete

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material behaviour presented in MC2010 [12] with fracture energy regularization. With this new concrete damage model for RC beams with RAC, researchers will have a tool to test new experimental campaigns.

1.2. Research significance

Initially, results from previous experimental campaigns were used [13], in which the tensile fracture energy for RCA was assessed using three point bending beam of plain concrete with notch in the middle. The outputs of the experimental campaign, will be used to compare and validate the new concrete damage, which uses fracture energy regularization, for both tensile and compressive behaviour. With the previous validation it is now possible to use a concrete damage model with crack band model regularization to numerically simulate RC beams with RCA. In this last case it will be used the results from the previous experimental campaign reported by [14].

2. Previous studies

The load-bearing capacity of RAC beams has been extensively investigated as a crucial mechanical attribute [9,15]. Several studies have shown that the bending strength of RAC beams is comparable to that of natural aggregate concrete (NAC) beams, with the flexural performance largely unaffected by the replacement ratio of natural coarse aggregate (NCA) [16–18].

However, other studies suggest that the bending strength of RAC beams is nearly equal to that of NAC beams when the NCA substitution ratio is less than 50%. Conversely, the bending strength of RAC beams can decrease significantly when the substitution ratio reaches 100% [19]. Regarding bending stiffness, there is a general consensus that the substitution ratio of recycled aggregate has a significantly greater impact on mid-span deflection than the reinforcement ratio [16,17,20,21]. This effect is also evident when natural fine aggregate (NFA) is substituted with recycled fine aggregate (RFA) [22,23].

Numerous experimental studies have been carried out to assess the material properties of concrete made with both natural coarse aggregate (NCA) and recycled coarse aggregate (RCA) [24,25]. The flexural performance of RCA concrete beams at the structural level has also been widely explored through various experimental investigations [16,26,27], though the results remain inconsistent. The structural behaviour of concrete beams with three different RCA contents 20%, 50%, and 100% was studied by Choi et al. [22]. Their results revealed that the moment capacity of the RCA beams was similar to that of conventional concrete beams. They also noted that the crack patterns of the RCA beams were similar to those observed in regular concrete beams.

An experimental study was conducted by Sunayana and Barai [28] using concrete beams made with 100% recycled aggregates at three different rebar ratios. Their findings showed that, at lower rebar ratios, the moment capacities of ordinary concrete beams and RCA beams were comparable.

Kang et al. [27] investigated RCA beams with large cross-sections using both normal and high-strength concrete. The RCA replacement levels reached up to 30% for high-strength concrete and up to 50% for normal-strength concrete. Test results indicated that RCA beams exhibited lower ductility, and a higher frequency of cracking compared to conventional concrete beams. In a separate study, Arezoumandi et al. [16] tested eight beams, four of which incorporated RCA. Their findings revealed that the crack spacing in RCA beams was smaller than that in traditional concrete beams. Moreover, RCA beams displayed greater ultimate deflection. Despite these differences, the ultimate moment capacities of RCA and conventional concrete beams were found to be comparable. Currently, several researchers have conducted preliminary numerical studies to simulate the behaviour of RCA, both with and without reinforcement [29–32]. However, some parameters such as fracture energy remained uncertain in these studies [14,29]. In addition, some of the concrete damage models employed did not incorporate

compressive fracture energy regularization, which is crucial for accurately simulating the post-peak response of reinforced concrete structures. In recent years, the use of nonlinear analysis in concrete design has increased significantly [33], bringing it closer to practical applications in the civil engineering industry. As a result, there is a growing need to evaluate existing models for their suitability in representing recycled aggregates.

3. Concrete damage model

This previous concrete damage model has been fully presented in previous works [34], and it is beyond the scope of this work to present the full detail of its formulation and the chosen damage evolution laws. Mazars used the classical Eq. (1) to describe the constitutive relation for non-linear concrete behaviour [35], where E_d is the local damaged stiffness matrix at a certain time step. Using the description in Eq. (2), where E_0 is the local stiffness matrix for the undamaged material, the matrix E_d can be computed. Since the behaviour of undamaged material is assumed to be isotropic linear elastic, the values of this matrix E_0 are constant.

$$\{\sigma\} = [E_d]\{\varepsilon\} \quad (1)$$

$$[E_d] = (1 - d)[E_0] \quad (2)$$

The damage evolution for both compressive and tensile behaviour is controlled by the positive principal stresses, ε_i . Mazars defines the equivalent strain, ε_{eq} , in this paradigm, which is determined using Eq. (3).

$$\varepsilon_{eq} = \sqrt{\langle \varepsilon_1 \rangle_+^2 + \langle \varepsilon_2 \rangle_+^2 + \langle \varepsilon_3 \rangle_+^2} \quad (3)$$

with

$$\langle a \rangle^\pm = \frac{1}{2}(a \pm |a|) \quad (4)$$

The scalar variable equivalent strain, ε_{eq} , and a scalar parameter, ε_{d0} , which defines the evolution of the elastic domain, both influence the loading function, f . The value of ε_{d0} is initially a material constant, but it is modified to $\varepsilon_{d0}(d) = \varepsilon_{eq}$ once damage has begun (Eq. (5)).

$$f(\varepsilon_{eq}, \varepsilon_{d0}) = \varepsilon_{eq} - \varepsilon_{d0}(d) = 0 \quad (5)$$

In terms of materials, the maximum equivalent strain level attained in the concrete's load history is measured by $\varepsilon_{d0}(d)$, which is a scalar variable. Unloading takes place and there is no change in the damage variable if $f < 0$. If not, the damage variable rises, and the new value for d is obtained by combining two independent and scalar variables, d_T and d_C , linearly (Eq. (6)):

$$d = \alpha_T d_T + \alpha_C d_C \quad (6)$$

There are several possibilities to compute the values of the parameters of α_T and α_C , but in this study, the assumptions proposed by [36] are used. For this it is necessary to first compute the principal effective stresses (Eq. (7)), which are later combined using classical Macaulay brackets (Eq. (8)), to identify the positive and negative stress directions. Later these are again extrapolated into strain vectors for tension and compression (Eq. (9)).

$$\{\hat{\sigma}\} = [E_0]\{\varepsilon\} \quad (7)$$

$$\{\hat{\sigma}\}_\pm = \frac{1}{2}(\{\hat{\sigma}\} \pm |\{\hat{\sigma}\}|) \quad (8)$$

$$\begin{aligned}\varepsilon_{T_i} &= \frac{1+\nu}{E} \langle \hat{\sigma}_i \rangle_+ - \frac{\nu}{E} \sum_k \langle \hat{\sigma}_k \rangle_+ \\ \varepsilon_{C_i} &= \frac{1+\nu}{E} \langle \hat{\sigma}_i \rangle_- - \frac{\nu}{E} \sum_k \langle \hat{\sigma}_k \rangle_- \end{aligned} \quad (9)$$

Eq. (10) can be used to calculate the coefficients d_T and d_C using only the positive components of the vectors ε_T and ε_C . This is because only positive strains cause damage, according to the Mazars model's initial postulate. These variables remain constant during a specific time step in order to encourage stabilisation during the convergence process.

$$\begin{aligned}\varepsilon_V^+ &= \sum_i \varepsilon_{T_i}^+ + \sum_i \varepsilon_{C_i}^+ \\ \alpha_T &= \frac{\sum_i \varepsilon_{T_i}^+}{\varepsilon_V^+}; \alpha_C = \frac{\sum_i \varepsilon_{C_i}^+}{\varepsilon_V^+} \end{aligned} \quad (10)$$

It is important to point out that this damage model formulation only accounts for the mode I fracture process. Although the constitutive relation is isotropic after the damage initiation, it is still possible to obtain partial non-linear shear behaviour due to the shear distortion being indirectly hidden in the principal strains. To take account of this effect indirectly, it is possible to calibrate some of the damage parameters [37].

4. Damage evolution

The Mazars damage model is typically used in conjunction with a regularisation technique that takes into account either gradient [38] or non-local integral [39] models in academic implementations. Though they are not widely used in the industry or in commercial FE software, these regularisation approaches are highly effective in guaranteeing the objectivity of the numerical solutions. This is because, anytime non-local regularisation is used, it is known to be difficult to do effective parallel computation with legacy solvers [40]. Furthermore, using gradient and integral non-local models may result in computationally demanding processes.

Compared to the other two regularisation methodologies mentioned earlier, fracture energy regularisation offers a less reliable approach. Despite this, the industry, the FIB's guidelines [41], and commercial FE software all use this kind of regularisation, which has no significant disadvantages when using parallel processing methods [42].

This work suggests additional damage evolution laws for tension and compression behaviour in order to support the usage of fracture energy regularisation for the Mazars damage model. All of the previously mentioned thermodynamic consistencies are retained in the original Mazars formulation. All the damage evolution laws are rebuilt in terms of the equivalent strain parameter, which is described by Eq. (3), in order to maintain the essential elements of the original Mazars loading function.

A classical exponential softening law is proposed for tensile behaviour. Fig. 1 a) illustrates how the initial non-linear behaviour resembles

the connection shown in MC2010 [12]. Branch OA exhibits linear stress-strain behaviour at first, followed by branch AB, which exhibits non-linear behaviour with damage. The development of damage is then modelled using an exponential softening rule (branch AB). The material behaviour commonly seen in uniaxial tensile strength testing is faithfully replicated by this model [39,43]. The following defines the tension damage evolution law:

$$d_T = 1 - \frac{\varepsilon_{d0}}{k_r \varepsilon_{eq}} \exp\left(\frac{\varepsilon_{t1} - \varepsilon_{eq}}{\varepsilon_{tu} - \varepsilon_{t1}}\right) \quad (11)$$

All parameters and physical quantities in Eq. (11) are specified as follows:

$$\sigma_{t0} = k_r f_{ctm}; \sigma_{tmax} = f_{ctm}; \varepsilon_{t0} = \varepsilon_{d0}; k_r = 0.9 \quad (12)$$

$$\varepsilon_{t1} = \max\{0.015\%; 1.1 \varepsilon_{d0}\}; \varepsilon_{tu} = \frac{G_{ft}}{L_c f_{ctm}} + \varepsilon_{t1}; g_{ft} = \frac{G_{ft}}{L_c} \quad (13)$$

While some of the variables are redefined, others are described in MC2010: L_c is the characteristic length of the element; G_{ft} and g_{ft} are the fracture energy and volume fracture energy of the material, respectively; f_{ctm} is the mean uniaxial tensile stress of concrete; ε_{tu} is the equivalent ultimate strain for bilinear softening and can be calculated using the tangent to the exponential softening law calculated at point A, as shown in Fig. 1 a). Half of the volume fracture energy is represented by the area denoted by triangle A- ε_{tu} - ε_{t0} , and the other half is represented by the area between the exponential softening curve and the tangent to the exponential softening curve calculated at point B.

MC2010 uses a bilinear law to determine softening under uniaxial strain. When damage gets close to 1.0, this could cause convergence issues. When the exponential softening law is applied, it guarantees the constitutive model's steady numerical behaviour.

The MC2010 stress-strain relation for uniaxial compression is modified for simulating compression behaviour. Up until the beginning of softening (branch OAB in Fig. 1 b), the MC2010 relation is applied. Although there is disagreement about what constitutes a suitable softening branch (branch BCDE) [45], the simplification presented in [41] is used here. A straightforward bilinear function is used to guarantee simplicity and encourage computational efficiency with constant stress in branch BC and falls off linearly in branch CD (Fig. 1 b).

In order to reduce convergence issues, the model used here admits a residual compressive stress, as recommended in fib bulletin 45 [41]. Point A is where damage starts, and it could get worse over the course of the loading history. The assumption given in [46] is used to calculate the volume fracture energy (g_{fc}). The corresponding ultimate compressive strain is calculated under these circumstances, ignoring the residual stress, as shown in Eq. (14).

$$g_{fc} = \frac{G_{fc}}{L_c} = \sigma_{cmax} \times \left(\frac{\varepsilon_{cu} + (\varepsilon_{c2} - \varepsilon_{c1})}{2} \right) \quad (14)$$

The law defining the evolution of compressive damage is:

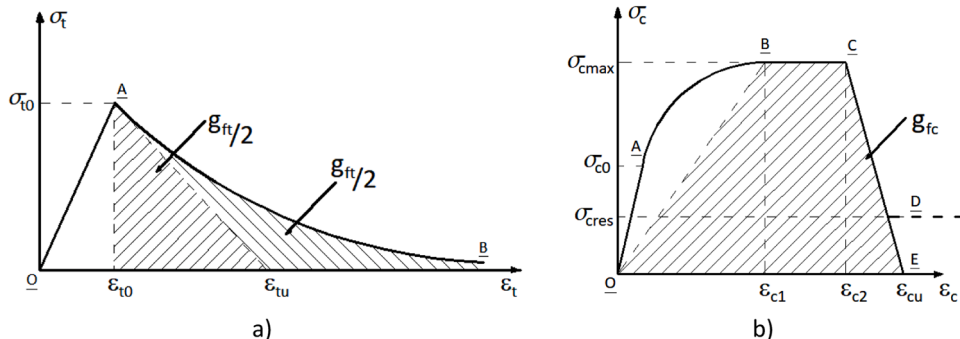


Fig. 1. a) tension vs equivalent strain and b) compression vs equivalent compression strain [44].

$$d_c = \begin{cases} 1 - \frac{(k \times \bar{\varepsilon}_c - \bar{\varepsilon}_c^2) f_{cm}}{(1 + (k - 2) \times \bar{\varepsilon}_c) E_{cm} \varepsilon_{eq,c}} & \text{if } \varepsilon_{eq,c} \leq \varepsilon_{c1} \\ 1 - \frac{f_{cm}}{E_{cm} \varepsilon_{eq,c}} & \text{if } \varepsilon_{c1} < \varepsilon_{eq,c} \leq \varepsilon_{c2} \\ 1 + \frac{k_1}{E_{cm}} - \frac{k_2}{E_{cm} \varepsilon_{eq,c}} & \text{if } \varepsilon_{c2} < \varepsilon_{eq,c} \leq \varepsilon_{cu} \end{cases} \quad (15)$$

where

$$\sigma_{max} = f_{cm}; \quad \varepsilon_{eq,c} = \frac{\varepsilon_{eq}}{\sqrt{2}}; \quad \bar{\varepsilon}_c = \frac{\varepsilon_{eq,c}}{\varepsilon_{c1}}; \quad k = \frac{1.05 E_{cm} \varepsilon_{c1}}{f_{cm}}; \quad k_1 = \frac{f_{cm}}{(\varepsilon_{cu} - \varepsilon_{c2})}; \quad k_2 = f_{cm} + k_1 \times \varepsilon_{c2} \quad (16)$$

The compressive damage evolution law (Eq. (15)) and the adopted variables (Eq. (16)) agree with the stress-strain constitutive relation for uniaxial compression presented in MC2010, despite the fact that some modifications were necessary due to the use of Mazars damage models in conjunction with the fracture energy regularisation. These equations use the following: f_{cm} is the concrete's mean compressive strength; E_{cm} is the secant Young's modulus; ε_{c1} and ε_{c2} are strain parameters taken from MC2010; k , k_1 and k_2 are parameters that describe the constitutive curve; $\bar{\varepsilon}_c$ is a unidimensional strain ratio given in MC2010; and G_{fc} and g_{fc} are the compressive fracture energy and the volume compressive fracture energy defined for the material, respectively.

5. Time step integration

Despite this damage model relying on the ABAQUS standard, its time step integration follows the methodology described in the work of [47, 48]. This algorithm is capable of automatically identifying convergence issues during specific time step increments by detecting repeated time step reductions. When such issues arise, it switches the time step integration scheme from an implicit to an explicit formulation. The algorithm, implemented through UMAT, maintains solution accuracy even with very small time steps. Once convergence is restored and the time step reductions are minimized, the method reverts to the implicit formulation. A detailed explanation of the algorithm's implementation is beyond the scope of this study, but further information is available in Arruda et al. [44].

6. Numerical testing

In the numerical analysis, the previous experimental campaign reported in [13,14] is used. It is beyond the scope of this work to present a fully detail description of the loads, geometry, support conditions and even tested material properties, but for further information, the following publications should be checked [13,14].

For the numerical study, partial and fully recycled concrete specimens were adopted, with a percentage of recycled fine aggregates of 30% (R30), 50% (R50) and 100% (R100). The concrete material properties of the reference concrete and the recycled ones are presented in Table 1 and Table 2 for both tested models, respectively. The variation of the material properties with the percentage of recycled aggregates is in accordance with the formulations proposed by [13]. For the RAC beam used [14], it is admitted the compressive strength was measured, but all

Table 1

Concrete material properties used in the numerical models for the three point bending beam with notch.

Name	E_{cm} [GPa]	ν [-]	f_{cm} [MPa]	f_{cm} [MPa]	G_{ft} [N/m]	G_{fc} [N/m]
R0	33.29	0.18	2.88	33.8	120.2	21218.4
R30	29.93	0.20	2.62	33.5	125.3	21171.1
R50	27.50	0.22	2.39	32.5	121.6	21011.3
R100	21.28	0.23	1.78	28.6	94.7	20350.5

Table 2

Concrete material properties used in the numerical models for the four point bending test in the RC beam.

Name	E_{cm} [GPa]	ν [-]	f_{cm} [MPa]	f_{cm} [MPa]	G_{ft} [N/m]	G_{fc} [N/m]
R0	32.32	0.18	2.64	30.64	109.0	20704.0
R30	28.51	0.20	2.26	28.48	106.5	20329.1
R50	25.89	0.22	1.99	26.56	99.3	19977.4
R100	20.52	0.23	1.59	25.36	84.0	19747.9

the other properties are estimated using the previous analytical expressions recommended in [13]. For the Poisson coefficient, variation follows the recommendations proposed by [49]. The adopted value of the tensile fracture energy G_{ft} was measured using a standard three point bending test with a notch, according to the specifications in TC50-FMC from RILEM-1985 [50]. According to the author's best knowledge, there is no consensus in the scientific community about the adopted values for compressive fracture energy G_{fc} . For this reason, due to the lower compressive resistance of recycled aggregate concrete, it was decided to use the expression (Eq. (17)) provided by [51] and tested numerically in the work of [34].

$$G_{fc} = 8.8 \sqrt[4]{f_{cm}} \quad (17)$$

All the mechanical properties of the steel rebars, are presented in Table 3, and it is admitted that the steel presents no softening branch, since in the experimental campaign, the values of measured strain in the rebars were below 0.3%, which were in accordance with [52].

All the mechanical properties presented in Table 1, Table 2 and Table 3 for concrete and steel rebars respectively, are used in all FE models during the numerical campaign for all four structural specimens. For all numerical tests, to assess that no mesh dependency [53] was detected, three meshes from coarse to fine were tested, in which the finer one was chosen for the final model validation. Numerical instabilities, convergence issues and even mesh dependency related to the new proposed UMAT was already studied in [44]. To ensure consistent result interpretation in the non-linear analysis, all models used plane stress FE elements (CPS4) with four nodes and 2×2 Gauss integration points. Due to the small observed displacement during the previous experimental campaign, a standard geometrical linear analysis was used to ease the convergence process. Also, due to high softening branches during the structural response in all models, imposed displacements are applied, since only classical Newton-Raphson is used, during the predictor-corrector incremental iterative process. To better understand the final equivalent crack distribution, the equivalent scalar strain variable $\varepsilon_{d0}(d)$, presented in Eq. (5) is shown along with final damage distribution.

6.1. Three point bending beam with notched

The mechanical behaviour of concrete depends, to a large extent, on properties that are not directly represented by the limiting capacity for a given type of load. In order to know the behaviour of structures until failure, allowing better numerical modelling of the phenomena, it is necessary to establish the response of concrete after peak load, analysing softening in the non-linear phase, allowing the global response of the material to be measured. For this reason, determining the fracture behaviour of concrete plays an essential role, allowing the establishment of more complex and reliable constitutive relationships for the response of structures produced with this material. A classical standard test is the three point bending beam with a notch at mid span. The geometry,

Table 3

Steel material properties used in the numerical models.

Name	E [GPa]	ν [-]	f_{sy} [MPa]	f_{su} [MPa]
Rebar	210	0.3	516	635

support conditions, and loading illustrated in Fig. 2 are based on the specifications of FTC50-FMC from RILEM-1985 [50]. At mid span, there is a notch, with a length that is a quarter of the beam height, as illustrated in Fig. 2, with a 3.0 mm opening, in which the CMOD was experimentally measured.

The load is centred at mid span, in which a metal plate is simulated in the FE model to better distribute the pressure stress in the top of the beam, as shown in Fig. 3 a). Due to the high stress tensile stress concentration at the bottom tip of the notch, that zone was heavily refined, as depicted in Fig. 3 b). To reduce the mesh refinement near the zones with low stress concentration, an advancing front mesh algorithm was used, varying the mesh size from 8.5 mm to 1.5 mm in the notch region. It was then possible to optimize the computational cost near zones that require higher accuracy. It was chosen not to use symmetry simplification, since some part of the fracture energy could be artificially dissipated in half of the notch if the mesh was not refined enough. Therefore, on the left of the beam, a pinned support is used, and a horizontal roller is inserted on the right side. In this numerical test, before the imposed displacement at mid span, the self-weight of the beam is applied as an imposed density load. This is necessary since the initial tensile stress at the notch change the maximum force due to imposed displacement.

Fig. 4. presents the level of equivalent strain variable that promotes the propagation of damage, for all four models. The image displays an increase scale deformation, in order to visualize the crack opening. It is also displayed the damage distribution around the notch zone, as presented in Fig. 5. Since the level of variation of the tensile fracture energy is not very different from the reference concrete, both recycled aggregate specimens presented the same final damage distribution. Regarding the observed equivalent scalar strain variable, which is associated with the level of possible crack opening, the R100 presented a slighter smaller value near the bottom of the notch, which is associated with a lower cracking tensile strength.

For the structural response observed in Fig. 6, all models presented the same variation between what was observed in the experimental and numerical campaign. In order to eliminate any deformation that may result due to different support conditions, it was chosen to plot only the crack mouth opening displacement (CMOD). In this case, the deviation between the maximum peak load is similar in both numerical models and experimental specimens. The R0 presents the higher load peaks and larger displacement just before the existence of softening. This is also observed for R30 and R50 but to a smaller extent. Only the numerical response of R100 presents a lower maximum peak load than the experimental one. This may suggest that the adopted values of the fracture energy and tensile stress may be overestimated. There is also a small difference related to the initial stiffness of the numerical models, which is later converted to a phased softening branch, providing extra energy in the numerical model. This was due to some of the supports in the experimental campaign presenting some horizontal friction, promoting some artificial horizontal force that was not presented in the numerical model.

6.2. Four point bending test in RC beam

The beam loading and support scheme is shown in Fig. 7, with a

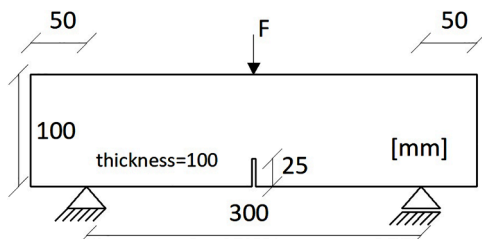


Fig. 2. Geometry for the test setup.

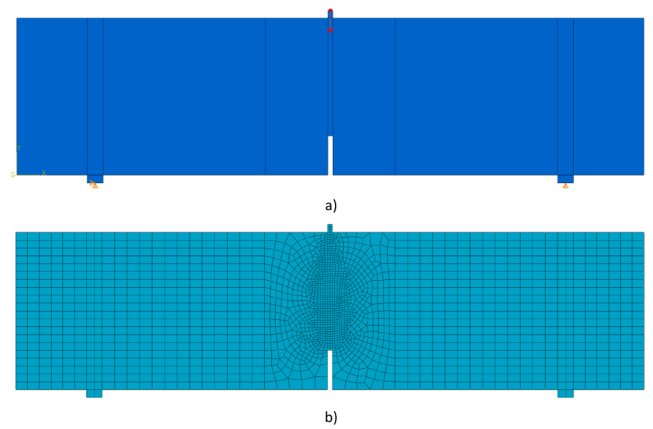


Fig. 3. a) Adopted load and support conditions for the numerical model; b) Used mesh in the three point bending beam.

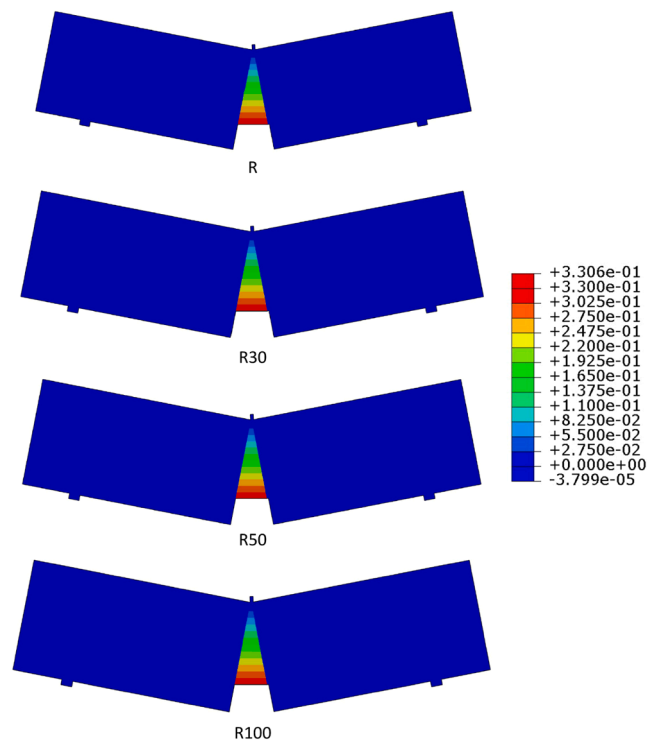


Fig. 4. Final equivalent scalar strain variable in the three point bending beam.

pinned and horizontal roller support at the left and right, respectively. The main advantage of the model used is the fact that the bending moment presents a zone with a constant value, where the transverse effort is null. For this reason, the authors of the original work reported in [14] chose not to use rebar stirrups in that zone. It is then possible to analyse the beam's response to bending (distance between cracks and respective evolution) without needing to consider the joint effect of the two types of stresses (bending and shear).

Initially, with some simplifications, the experimental values were directly compared with the analytical models to assess and compare the accuracy with the numerical campaign below. Considering the small dimensions of the beam span, the authors chose steel rebar that suited the scale of the beams without producing any unexpected scale effects. In any case, the conclusions about the behaviour of the recycled aggregates can be extrapolated for larger real beams.

According to the description reported in [14], the data acquisition during the tests was carried out using a set of sensors that allowed the

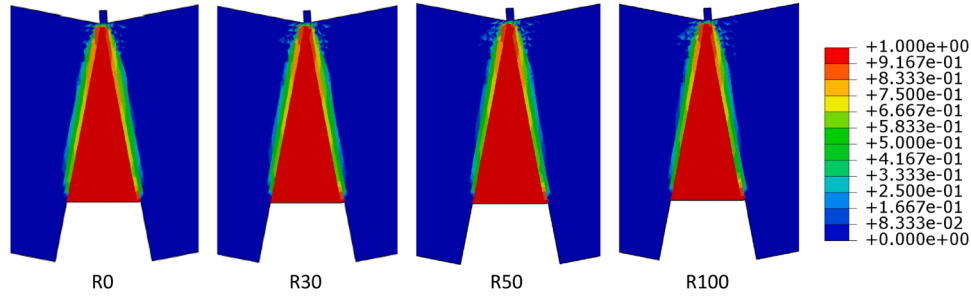


Fig. 5. Final damage distribution in zoomed at the notch region.

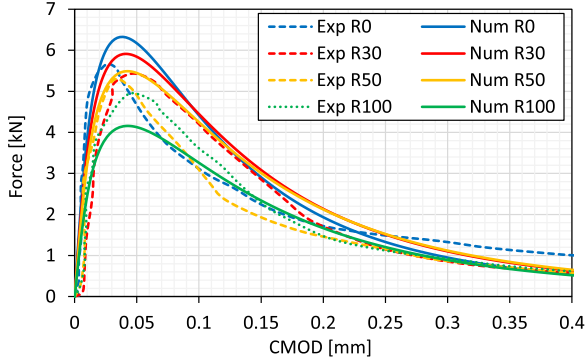


Fig. 6. Force vs displacement for both the experimental and numerical models for the three point beam with notch at mid span.

beam's behaviour to be measured comprehensively. The displacements were read at three points (aligned with the loads and in the middle span of the beams) using three deflectometers with a maximum stroke of 50 mm.

During the experimental campaign, some unexpected results occurred, and this is fully reported in [14]. On the one hand, it would be expected that the distance between cracks would be smaller for concretes with a higher percentage of recycled aggregates, as they have lower tensile strength and, therefore, a smaller distance would be needed to mobilize the cracking stresses, as indicated in numerous classic references for the design of reinforced concrete structures [54]. On the other hand, a possible lower adhesion between steel and the reference concrete R0 [24,55] would require a greater distance to mobilize the same force in the concrete. Although the numerical models were not capable of actually measuring the crack openings, these are proportional to the equivalent scalar strain. This equivalent scalar strain is indeed similar in all four damage models, as shown in Fig. 8, indicating the possible crack openings being also very similar between the reference concrete and the ones with recycled aggregates. In any case, for R50 and R100, the level of expected crack propagation due to higher equivalent scalar strain dispersion is greater, according to the numerical models. This may suggest that larger crack openings are expected for service loads.

Regarding the structural response in terms of force vs mid span displacement, there are some minor differences between the experimental and numerical results, as depicted in Fig. 9. First, the numerical model shows a higher force before the transition from the uncracked (state I) to fully cracked (state II), although the post crack stiffness is the same in the experimental and numerical model. This, as previously suggested, may indicate that the analytical formulas used to estimate tensile strength and tensile fracture energy are overestimating the real values. Second, the drop due to compressive crushing in the top surface that occurs in the experimental campaign after 60 mm of displacement, is more pronounced in the experimental campaign. Only for R100 it is observed in Fig. 8 some crushing-related equivalent scalar strain in the top surface near the applied load. In all other R0, R30 and R50 this is also observed in the same spot but at a smaller scale, not being represented by a large force drop in Fig. 9. In any case, for R100, a force drop in Fig. 9 is observed during the structural response, approximately at the same time as in the experimental campaign. This might suggest that the expressions used for the compressive fracture energy are also overestimated.

7. Conclusions

In this study, a new damage model based on the constitutive relations proposed by model code 2010, was used to simulate the behaviour of concrete structures with recycled aggregates. All numerical models exhibited structural responses and distributed equivalent scalar strains similar to those observed in the experimental campaign.

- The adopted values, derived from previous analytical formulas for the estimated tensile strength and tensile fracture energy, led the numerical models to show higher deformation energy in the structural response, and in some cases higher maximum peak forces. This may suggest that the previous formulations for the material properties adopted to model the structural behaviour of concrete structures with recycled aggregates are indeed overestimation.
- Regarding the compressive fracture energy, which was indirectly estimated, using the same expressions as in natural concrete, did not consistently exhibit the expected concrete crushing, in contrast to the experimental campaign. This issue could be explored further in future studies, where a more accurate general expression could be

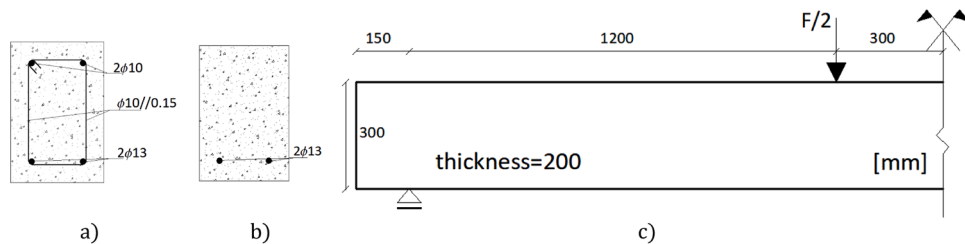


Fig. 7. RC beam with four point bending: a) cross section between support and load; b) cross section between loads; c) test setup.

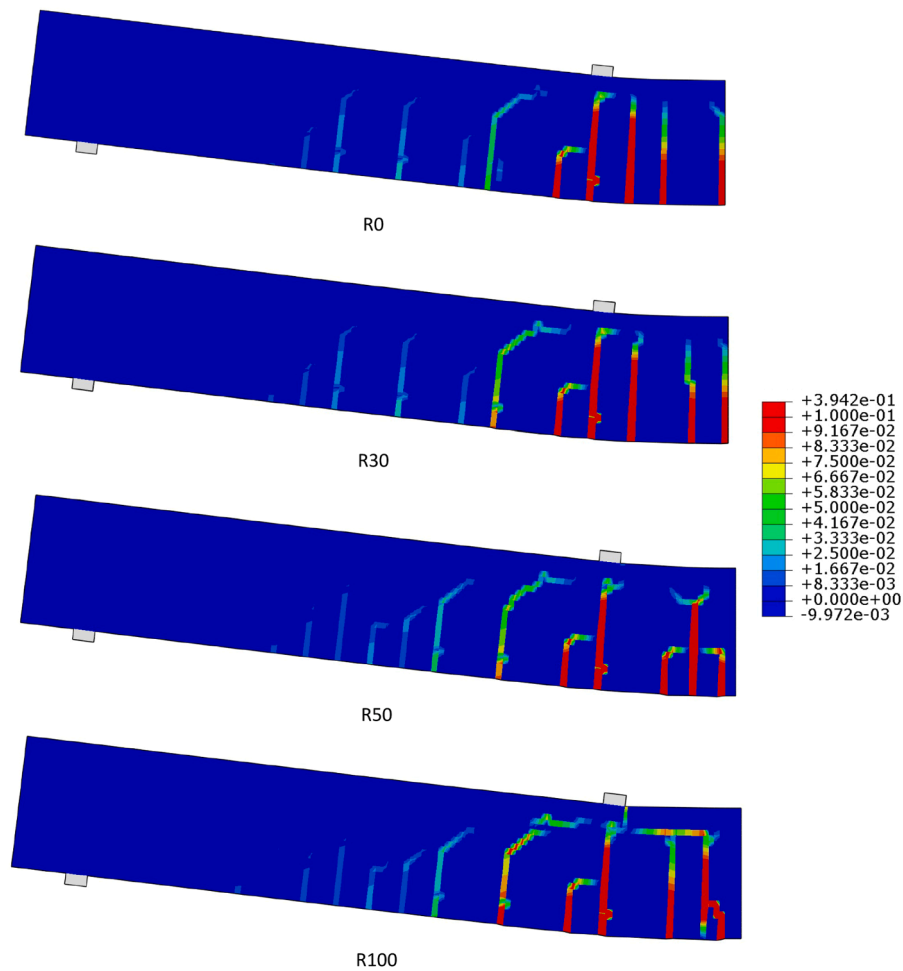


Fig. 8. Final equivalent scalar strain variable in the four point bending RC beam.

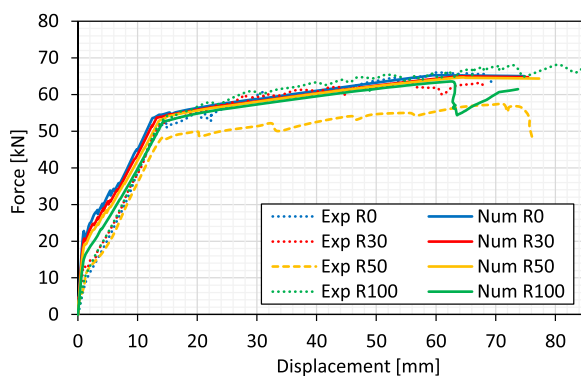


Fig. 9. Force vs displacement for both the experimental and numerical models for the four point reinforced concrete beam.

developed to estimate compressive fracture energy for recycled aggregates.

- Regarding the possible crack distribution and indirect possible crack opening due to higher values of the equivalent scalar strain variable, the concrete with a higher percentage of recycled aggregates is expected to perform poorly according to the results presented by the numerical model. This again should be improved in the future, with more experimental campaigns with beams with higher spans.

Data availability

All data and UMAT files will be publicly displayed and ready to be downloaded by the scientific community on GitHub at github.com/marioruiarruda/.

CRediT authorship contribution statement

M. Zitouni: Writing – review & editing, Visualization, Investigation, Data curation. **M.R.T. Arruda:** Writing – original draft, Validation, Supervision, Software, Project administration, Investigation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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