

Research papers

A semi-analytical model for the velocity profile in an open channel with suspended rigid vegetation

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ABSTRACT

Aquatic suspended vegetation increases a significant flow resistance at the water surface, resulting in a hyperbolic-type velocity distribution in the gap between the vegetation and bed, similar to the flow in a channel covered with ice sheets. To depict the hydraulic mechanism from the bed to the free surface in both the vegetated and non-vegetated portions of the suspended vegetation flow, the four interconnected zones of flow—the interior vegetation layer, vegetation shear layer, near vegetation layer, and bottom boundary layer—must be modeled. Based on the mixing-length concept, this study proposed a novel semi-analytical model to predict the vertical velocity distribution in the open channel flow covered with suspended rigid vegetation. The shear stress in the interior and non-vegetation region was calculated using the mixing length theory, and a few parameters were established using Plew's extensive laboratory tests and published data. The proposed model makes good predictions with data for various vegetation conditions. These findings show that the suggested model can accurately predict the vertical velocity profiles in the case of suspended vegetation. Based on five parameters—the drag coefficient, mixing length constants, penetration depth, and the location of the highest velocity—the current results demonstrate that the zone-wise suggested model may be utilized to forecast the vertical velocity distribution in suspended vegetation.

1. Introduction

Studying suspended, submerged, and emergent vegetation distributed in streams and wetlands is essential in understanding the flow hydrodynamics in water ecosystems. The submerged and emergent category can be classified as vegetation that extends from the bottom surface to and beyond the free surface in a river channel (Yager and Schmeckle, 2013). However, the suspended vegetation has unanchored roots floating in the water, and the stems, leaves, and flowers are above the water surface. Like suspended aquatic vegetation, macrophytes play an essential role in ecosystem functions such as trophic interactions, biomass cycling, wastewater purification, and ecosystem structuring (Zhang and Nepf, 2011; Li et al., 2020). The global abundance of suspended vegetation such as water hyacinth (*Eichhornia crassipes*), which often forms choke points due to accumulation, is evident from the fact that these plant masses can be detected from space (Coetzee et al., 2017). Reservoirs are particularly vulnerable to *Eichhornia crassipes*, one of the world's most invasive weed species (Kleinschroth et al.,

2021). Other species of suspended vegetation, like *Pistia stratiotes* L., *Salvinia molesta* D. Mitch, etc., can grow under appropriate conditions in any aquatic ecosystems (Gui et al., 2024). These suspended macrophytes have several drawbacks and advantages for the ecosystem. For example, *Eichhornia crassipes*, found in Brazil, Southeast Asia, and Europe, absorbs solar radiation and oxygen that would otherwise support other components of the local ecosystem (Van Driesche and Abell, 2008; Gui et al., 2024). There are positive aspects of the suspended vegetation as well since *Eichhornia crassipes* is also a rich source of biogas and absorbs organic and heavy materials like nitrogen, phosphorus, and lead (Todd and Josephson, 1996; Meerhoff et al., 2003; Hubbard et al., 2004; Padial et al., 2009). In addition to their ecological elements, macrophytes impact the velocity distribution, flow structure, and the movement of sediments and contaminants. This study focuses on the vertical distribution of streamwise velocity, which creates intricate profiles beneath and inside the suspended vegetation flow. The suggested model will help assess the flow dynamics of suspended vegetation, such as water hyacinth and other floating species, in open channels.

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This can support further studies on the velocity relationship with vegetation trapping and accumulation, which obstructs boat traffic, reduces irrigation capacity, and interferes with hydropower generation.

The literature contains experimental studies of free surface flows under submerged or emergent flow conditions in natural watercourses with varying plant height, density, and porosity (Singh et al., 2024; Tang et al., 2019; Tang et al., 2021; Tang et al., 2023a,b). Earlier studies of single-layered vegetation with different densities, diameters, and configurations have revealed inflections in the vertical streamwise velocity distribution by examining the flow patterns and severe shearing across the edge of the submerged vegetation (Raupach et al., 1996; Liu et al., 2008). To comprehend the flow dynamics, the flow and hydraulic resistance provided by the vegetation arrays have also been investigated concurrently (Stone and Shen, 2002). Additionally, efforts have been made to use the mixing length hypothesis to simulate the velocity flow distribution in emergent and submerged rigid vegetation in order to estimate and assess how well such models function in various configurations and densities (Tang, 2019; Wang et al., 2022). Some efforts have extended to double- or multi-layered canopies to explore the combined influence of different density layers on flow dynamics (Liu et al., 2010; Huai et al., 2014; Rahimi et al., 2020a,b). In the present study, we focus exclusively on a single-layer suspended canopy to isolate and better understand the effects of vegetation-induced blockage in the horizontal direction. Suspended vegetation, unlike submerged vegetation, introduces distinct flow characteristics due to its partial submergence—specifically, the gap between the vegetation and the channel bed—along with its floating nature, which complicates both flow measurement and numerical modeling.

Nevertheless, experimental investigations of suspended vegetation have shown that the flow field becomes complex, exhibiting altered vertical profiles of streamwise velocity and Reynolds shear stress both within and beneath the vegetation canopy (Plew, 2011; Fu et al., 2020; 2021; Gui et al., 2024). For instance, Plew (2011) used rigid cylinders to study the flow structure beneath and through suspended canopies. The experimental findings of Plew (2011) indicated that the distance between the vegetation and the bottom boundary layer limits the shear layer's ability to penetrate the vegetation. Thus, submergence of the suspended canopies defines the vertical velocity layering. The shear and density within the internal vegetation layer influence the flow field by enhancing vertical mass and momentum exchange, thereby impacting hydraulic properties such as velocity, energy transport, and turbulent diffusivity. The shearing split the vegetation layer into several zones of fast and slow transport after penetrating a considerable distance (Nepf and Ghisalberti, 2008). Conversely, in suspended vegetation, the bottom boundary limits the expansion of the shear layer and restrains the eddy scales. Incorporating the shear layer inside the suspended vegetation, where the channel bed effect is also present, is crucial for analytically modelling the flow-altered velocity profiles in the suspended vegetation.

Flow through suspended vegetation can be investigated using experimental measurements or simulated numerically through computational fluid dynamics (CFD) techniques. Previous experimental (Plew, 2011; Fu et al., 2020; Zhang et al., 2023) and numerical (Zhao et al., 2017) studies have rigid cylinders mimicking the suspended vegetation. These experimental studies have observed the vertical profiles of velocities, Reynolds stresses, turbulent shear stresses, and spectral density at the mid-width of the flume in the fully developed flow region. Fu et al. (2020) investigated the flow events and coherent structures in the suspended vegetation. Gui et al. (2024) used mature *Eichhornia crassipes* to simulate a real scenario in a natural environment in summer. However, Plew (2011) datasets were pivotal in all these studies since it is the most used data for developing the analytical models for the vertical distribution of streamwise velocity for the flow with the suspended vegetation. Plew (2011) performed 6 m-long and 0.6 m-wide flume experiments using particle tracking velocimetry (PTV). The floating vegetation occupied the entire test section in the flume. Model vegetation was constructed of aluminium cylinders with a diameter of $d = 0.95$

cm, and the frontal area per vegetation volume was $a = 1.0 \sim 1.9 \text{ m}^{-1}$. The relative flow depth was constant ($h_g/H = 0.5$), where H is the flow depth and h_g is the gap between the vegetation and the bed. For these test cases, the channel bed friction coefficient was estimated from $C_f = \tau_b / \rho U_b^2$, where the bed shear stress τ_b was estimated from the vertical profile of the Reynolds shear stress close to the channel bed. Despite these findings, experimental studies using suspended vegetation are difficult to conduct using intrusive methods like Acoustic Doppler Velocimeters (ADV) and with suspended obstruction over the open channel flows. In conclusion, the complex flow interaction with suspended vegetation is extremely laborious to acquire and process due to the experimental setup. Moreover, capturing the effect of vegetation flexibility—which significantly alters turbulence and flow resistance—is particularly challenging due to the dynamic nature of flow-vegetation interactions.

Besides these experimental studies, investigators also explored numerical simulations to study the suspended vegetation. To simulate exchange flows between open water and floating vegetation, Liu et al. (2017) developed a modified RNG $K-\varepsilon$ turbulence model in OpenFOAM. Zhao et al. (2017) have implemented the $K-\varepsilon$ turbulence model with the FLOW-3D module to obtain Reynolds stress and mean velocity profiles of open channel flow. Because of the combined action of suspended vegetation and channel bed roughness, asymmetrical flows are generated vertically due to suspended vegetation. To a certain extent, flow characteristics with suspended vegetation are similar to ice-covered flow characteristics (Han et al., 2018). However, when the computational grid has a resolution corresponding to the diameter and size of each cylinder, the computational cost of the numerical method becomes an issue when analysing vegetative flow movement with arrays of cylinders (Chang et al., 2017). The computing grids are bound by the dimensions of the individual cylinders so that the grid resolution is proportional to the size of each cylinder. Despite the fact that numerical simulation validates the mean flow through vegetation in the compound open channel, it takes a lot of time and computation power. Therefore, a semi-analytical solution using the double-averaged momentum equation as a control equation can be calibrated and tested to simulate the averaged streamwise velocity in the suspended vegetation, which the field engineers can use. This motivated the present study to develop a semi-analytical model of the velocity profile in a channel with suspended vegetation.

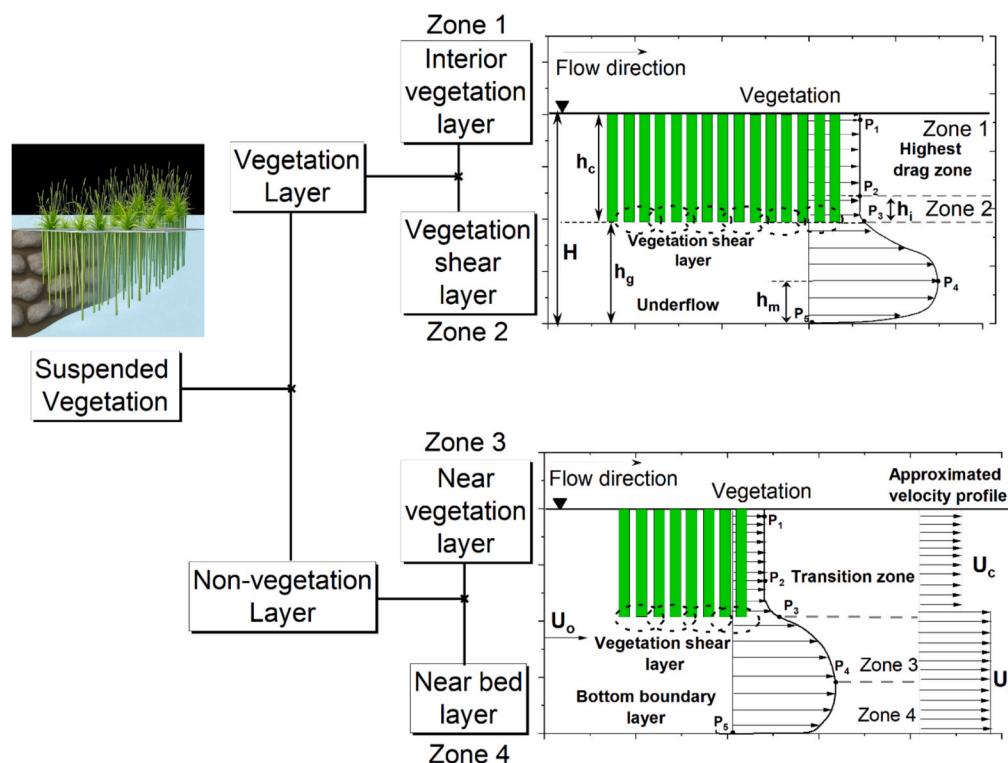
Previously, Zhang and Nepf (2011) developed a model by simulating the root layer with partial-depth porous obstructions, investigating its influence on vertical velocity and flux exchanges between free water and the vegetation layer, and predicting the magnitude of exchange flow based on energy considerations. Later, Huai et al. (2012) used Plew's (2011) experimental data to create a three-layer analytical model that predicted the vertical distribution of Reynolds stress and flow velocity. The bottom boundary, vegetation shear, and interior vegetation layers were the vertical divisions of the flow fields used in their investigation. Huai et al. (2012) developed a model with layer-dependent assumptions that were analytically solved in four distinct zones. The range of values for the mixing length constants (similar to Karman's constant) in this model was critical and revealed certain uncertainty, along with the required position of maximum velocity via a trial-on-error approach. The results demonstrated poor prediction when applied to different test scenarios where the same value of Karman's constant is used. O'Donncha et al. (2015) used an enhanced 3-D hydro-environmental model to simulate the impact of suspended canopies on vertical flow structures. The vegetation impacts on the turbulence intensity and length scales could be described using their two-equation turbulence closure model, which used the Mellor-Yamada technique. Nevertheless, their studies' parameterization for the coefficient of drag calibration was complicated and tedious, and the model once again had high processing requirements.

Despite growing ecological and hydraulic importance, suspended

A new semi-analytical model based on the mixing-length concept is proposed to predict the vertical velocity distribution in open-channel flow with suspended rigid vegetation. Experimental data from the literature was used to validate the model, and the results showed that it agrees well with data from different vegetation densities. A three-layer semi-analytical model has been built using momentum balance over the upper vegetation layer, the vegetation transition layer, and the flow through the non-vegetated layer (see Fig. 1). In an open channel with suspended vegetation, the water body was subject to the following forces: gravity, turbulent shear stress, and vegetation-induced drag force. Shear stress can be overlooked for the outer vegetation layer (zone I); each force is important for the internal vegetation layer (zone II); no drag force from vegetation is taken into account for the non-vegetated layer, which is further separated into near vegetation layer (Zone III) and near-bed layer (Zone IV), as the former is believed to be influenced by the vegetation and the latter by the channel bed. The momentum equations in an open channel with suspended vegetation can be solved analytically to predict the vertical distribution of the streamwise velocity. The mixing length constants used in the suggested model are more precisely specified for the suspended vegetation. We found out that the mixing length constant in the transition zone is proportional to the

2. Theoretical background

Seven test cases from [Plew \(2011\)](#) and [O'Donncha et al. \(2015\)](#) were considered in the analysis, as shown in [Table 1](#). Models were calibrated and tested using these results from seven experimental runs (see [Table 1](#)), and the results were compared to those of [Huai et al. \(2012\)](#). These runs are differentiated based on the projected area per unit volume ($\alpha = 0.9 \sim 1.9 m^{-1}$), representing the whole density range from [Plew's \(2011\)](#) experiments. The distance between the bed and



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Table 1

Summary of test runs B2–B15 of [Plew \(2011\)](#) experimental analysis; H is the water depth; h_g is the gap between the vegetation and the bed; L_x and L_y is the streamwise and transverse spacing between vegetations, a is the projected area per unit volume inside the vegetation; Q is the flow rate; U_c and U_b are averaged velocities within and beneath the vegetation; C_D is the averaged cylinder drag coefficient over vertical direction; and C_b is the bottom friction coefficient.

Run	$Q(L/s)$	$H(mm)$	$U_o(m/s)$	$L_x(mm)$	$L_y(mm)$	$h_c(mm)$	$h_g(mm)$	$a(m^{-1})$	C_D	C_b	$U_c(m/s)$	$U_b(m/s)$
B2	7.1	200	0.059475	150	50	175	25	1.272	0.78	0.0029 ± 0.0002	0.0567	0.0789
B5	7.8	200	0.065725	150	50	150	50	1.272	0.64	0.0027 ± 0.0001	0.058	0.0889
B9	8.9	200	0.074375	150	50	125	75	1.272	1.02	0.0027 ± 0.0002	0.0566	0.104
B12	10.5	200	0.0882	100	50	100	100	1.908	1.01	0.0035 ± 0.0002	0.0604	0.116
B13	10.1	200	0.0848	150	50	100	100	1.272	1.07	0.004 ± 0.0003	0.0616	0.108
B14	10.1	200	0.0849	200	50	100	100	0.954	0.85	0.0031 ± 0.0001	0.0658	0.104
B15	10.3	200	0.0863	200	100	100	100	0.477	0.58	0.0025 ± 0.00009	0.0752	0.0974

vegetation also varies depending on the flow depth and vegetation height. In each calibrated experimental run considered in this study, the cylinders were organized in different longitudinal and transverse spacing to account for variations in the spacing and arrangement of the suspended vegetation. [Table 1](#) also reports the average velocities over the canopy depth (U_c) and the gap under the canopy (U_b). The values of U_c and U_b are key quantities that must be determined from the semi-analytical model to assess the new model's overall performance. It will be easier to comprehend the model performance in two separate areas of the suspended vegetation, namely the vegetated and non-vegetation zones, because the model was created for different zones of flows with averaged values of U_c and U_b .

This section illustrates the corresponding flow mechanism for assessing the flow characteristics around the suspended vegetation in open channel flow. It also describes important parameters for modelling the streamwise velocity using a double-averaged equation in the streamwise direction used as the control equation.

2.1. Flow mechanism in suspended vegetation

In a cross-section of suspended vegetation, there are two types of flow sections with distinct mechanisms that interact with each other: vegetated and non-vegetated. [Fig. 1](#) shows the flow zones, which can be further divided into an interior vegetation layer (Zone 1 with the highest drag), a vegetation shear layer (Zones 2 and 3), and a bottom boundary layer (Zone 4). The zones are identified based on velocity and turbulence profiles. Each of these regions is covered separately in further detail.

Three regions can be seen in profiles of the normalized streamwise velocity in [Plew \(2011\)](#), [Fu et al. \(2021\)](#), and [Gui et al. \(2024\)](#): velocities rising above the bed that peaks somewhere between the vegetation and the bed (zones 4 and 3); velocities falling into and within the vegetation (zone 2); and velocities in the upper vegetation of zone 1 that are close to a constant value. Near the boundary of the suspended vegetation in all velocity profiles, an inflection point was detected by a shift in the sign of the flow gradient caused by the vegetation shear layer. It is necessary to understand the important parameters required to analytically model the flow in each zone from the constant velocity region to the inflection point and then the falling limb of the profile due to boundary shear stress.

According to the schematic diagram ([Fig. 1](#)), there are five important sites for the zone-wise analytical modeling of the multi-mechanism of flow under suspended vegetation, which are P_1 to P_5 . Similar to the [Huai et al. \(2012\)](#) model, these critical points are at the water surface (P_1); the interface between the outer and internal vegetation layers (P_2), where the vertical velocity starts to change quickly; the interface between the vegetated and non-vegetated layer (P_3); the point P_4 where the longitudinal velocity reaches its maximum over the entire water depth; and the bottom of the channel (P_5). The vegetation layer height is denoted by h_c , the internal vegetation layer height by h_i , and the height of a point P_4 above the channel bed by h_m . In order to present the solution of the double-averaged momentum equation and determine the longitudinal velocity in various zones, it is critical to estimate the important parameters at each of these sites. The proposed mathematical model

requires an estimate of the drag coefficient C_D , h_i , h_m , and three proportionality mixing length constants. The subsequent sections discuss and suggest these parameters to aid the user in applying the proposed mathematical model.

2.2. Analytical models for the drag coefficient

Numerous studies have discussed how to calculate vegetation drag coefficients ([Wu et al., 1999](#); [Carollo et al., 2005](#); [Larmaei et al., 2009](#)). In order to calculate the drag coefficient, [Schlichting and Gersten \(1979\)](#) used:

$$\begin{aligned} C_D &= 3.07Re_p^{-0.168}, Re_p < 800 \\ C_D &= 1.0, 800 \leq Re_p < 8000 \\ C_D &= 1.2, 8000 \leq Re_p < 10^5 \end{aligned} \quad (1)$$

where Re_p indicates the typical Reynolds number given by $Re_p = UD/\vartheta$, in which ϑ is the fluid's kinematic viscosity. However, the drag coefficients determined using Schlichting's method were relatively consistent (ranging from 1.00 to 1.04), indicating that the drag coefficient is independent of Re_p for the conditions being studied. The drag coefficient formula is obtained by regression analysis of [Plew \(2011\)](#) experimental data ([Huai et al. 2012](#)), as shown by this:

$$C_D = 1.33\sqrt{ah_c}\left(\frac{H}{h_c}\right) \quad (2)$$

where (H/h_c) represents the influence of relative submerged vegetation depth and $\sqrt{ah_c}$ represents the effect of vegetation density. The [Huai et al. \(2012\)](#) regression model of C_D on the [Plew \(2011\)](#) data appears to perform the best for the analytical solution, according to the application of their models on some selected cases.

[James et al. \(2016\)](#) proposed a model for the interaction between the projected area of the vegetation per unit volume, cylinder diameter, and the suspended vegetation drag coefficient. The following Eq. (3) was the best-fit model to express the bulk drag coefficient of a cylinder array:

$$C_D = C_{sd}(2.1 - 67.7ad) \quad (3)$$

where C_{sd} is the drag coefficient of a single, isolated cylindrical dowel (user-defined parameter, a is the projected area of the vegetation per unit volume (m^{-1}), and d is the diameter of the cylinder.

Using the velocity in the upper vegetation and the estimated free-surface gradient, the drag coefficient for cylinders inside the internal vegetation layer was calculated by [Plew \(2011\)](#). In the study by [Plew \(2011\)](#), C_{sd} averaged 0.62 (standard deviation of ± 0.14), showing no significant patterns in either gap height or vegetation density. At the same Reynolds numbers and vegetation densities, these values are greater than the C_{sd} , about 0.43 to 0.48 values that are predicted deep under submerged vegetation by [Ghisalberti and Nepf \(2004\)](#). It should be noted that the vegetation's vertical average velocity determines the canopy drag coefficient in the analytical model. The difference between the C_D values derived from the previously mentioned Eqs. (2) and (3) are depicted in [Fig. 2](#), showing that Eq. (2) outperforms the model by [James et al. \(2016\)](#). Thus, Eq. (2) was used further to estimate the C_D value in

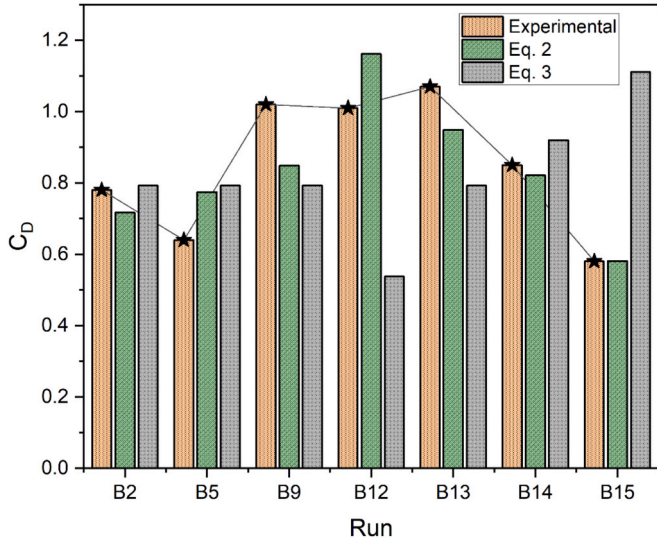


Fig. 2. Model performance of the vegetation drag coefficients C_D using different regression models given by Huai et al. (2012) and James et al. (2016).

the Huai et al. (2012) model application for the interior vegetation layer (Zone 1 with the highest drag) and a vegetation shear layer (Zones 2 and 3). The poor performance of Eq. (3) stems primarily from its empirical nature and the limited applicability of C_{sd} across varying vegetation configurations.

3. Model development

In open-channel flow with submerged and emergent vegetation, the drag from the vegetation is significantly greater than the shear friction on the bed and wall (Nepf and Vivoni, 2000; Stone and Shen, 2002; Tang et al., 2023a,b). Thus, the governing equation for fully developed vegetated flow can be described by the streamwise momentum equation of steady uniform flow (Klopstra et al., 1996; Baptist et al., 2007; Defina and Bixio, 2005; Ghisalberti and Nepf, 2004; Huai et al., 2014; Kubrak et al., 2008; Tang, 2018, 2019):

$$\frac{1}{\rho} \frac{\partial \tau(z)}{\partial z} = F_v - gS_o \quad (4)$$

where τ is the turbulent shear stress, z is the vertical coordinate above the bed, ρ the density of water, g is the gravity, S_o the bed slope and F_v is the drag force per unit mass generated by the vegetation (Fig. 3). The drag force F_v can be expressed as:

$$F_v = \begin{cases} \frac{1}{2} C_D a u^2, & z \leq h_c \\ 0, & z > h_c \end{cases}; a = m A_v \quad (5)$$

where C_D is the drag coefficient of vegetation, u is the streamwise time-average velocity, h_c is the suspended vegetation height, a the vegetation's density, i.e., the frontal area of vegetation (A_v) per unit volume, and m is the number of vegetation per unit area.

The turbulent shear stress can be described as follows using the Boussinesq hypothesis and the mixing length idea of eddy (Schlichting and Gersten, 2016):

$$\tau(z) = \rho \nu_t \frac{\partial u}{\partial z} = \rho l_v^2 \frac{\partial u}{\partial z} \left| \frac{\partial u}{\partial z} \right| \quad (6)$$

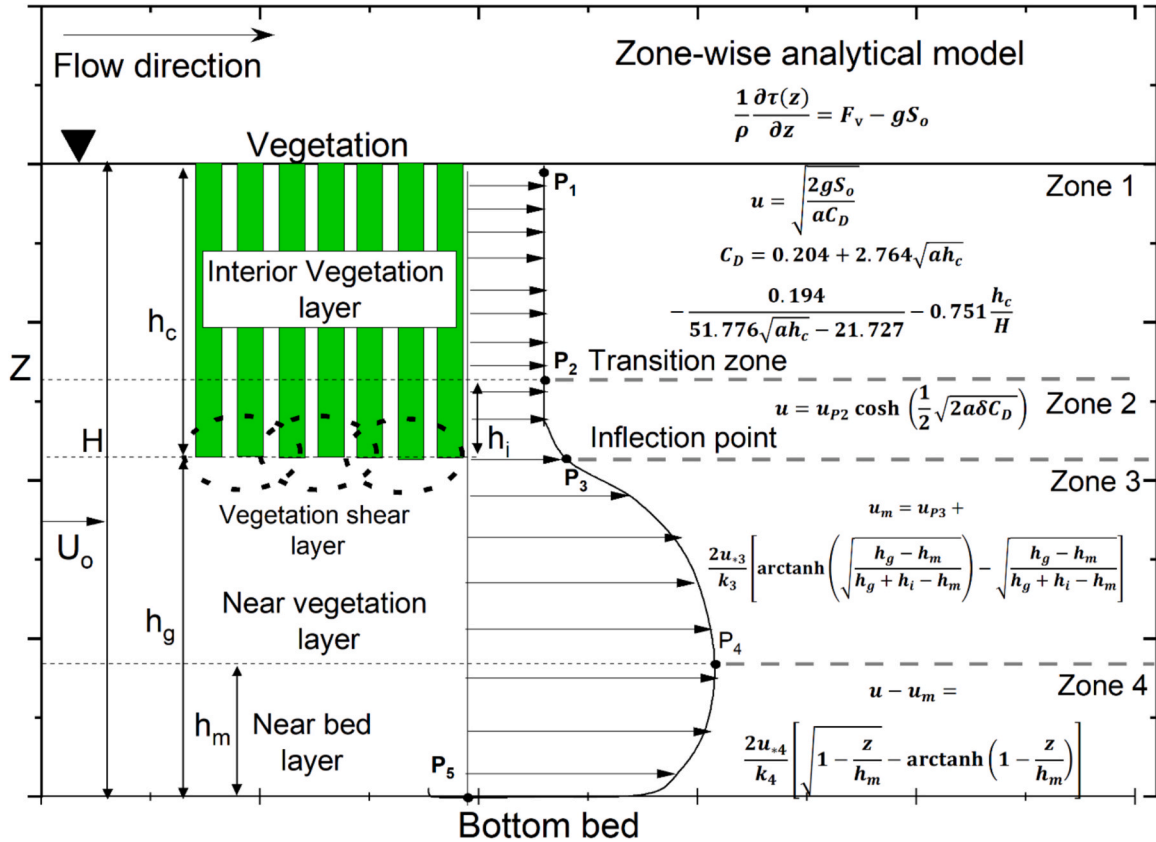


Fig. 3. An outline of the crucial parameters and developed model equations needed to generate the mathematical model for each zone, along with an illustration of zone-wise critical points.

where ν_t and l are the total eddy viscosity of vegetated flow and mixing length of the eddy, respectively.

3.1. Interior vegetation layer with the highest drag: Zone 1 (P_1 to P_2)

The drag force of the vegetation dominates the flow in this zone. Over this area, the vertical streamwise velocity variation is nearly constant for a given flow depth (Tang et al., 2023a,b). The velocity in this area is further reduced by increasing the flow resistance through increased vegetation density. As a function of vegetation thickness, the longitudinal velocity is influenced by the interior vegetation layer; higher velocities within the canopy result from a smaller gap between vegetation (Plew, 2011). Predicting the velocity distribution in the upper layer of vegetation heavily relies on the frontal area and the number of stems per area. Considering the shear stress on the free surface and the turbulent shear has minimum effect on the flow in this layer, Eq. (4) can be written as:

$$F_v = gS_o \quad (7)$$

Replacing F_v in Eq. (7) with Eq. (5) gives the expression for velocity within the vegetation layer, as previously described by Huai et al. (2012) and Tang et al. (2023a,b):

$$u = \sqrt{\frac{2gS_o}{aC_D}} \quad (8)$$

3.2. A vegetation shear layer: Transition zone 2 (P_2 to P_3)

An analogous definition of the transition layer or zone of inflection is flow through submerged vegetation. As seen in Fig. 3, the point of inflection in the vertical velocity profile is caused by the severe shearing at the height of the vegetation produced here. It is impossible to ignore the collective responsibility of Reynolds stress and drag forces in this layer of flow interaction of gravity. Another crucial flow feature in this layer, as seen experimentally by Plew (2011), is that when density declines, so does the Reynolds shear stress gradient beneath the vegetation layer and the stress at the vegetation height. The trend of the Reynolds stress illustrates a decreasing trend away from the bed, reaching a minimum till the height where maximum streamwise velocity is obtained and then increasing as it approaches the vegetation height, after which it asymptotically decreases till the free surface. Similar to Huai et al. (2012)'s expression:

$$\frac{du}{dz} = \frac{\sqrt{\frac{1}{2}a\beta C_D u^2 - gS_o}}{k_2 \sqrt{\delta}} \quad (9)$$

where $\delta(=h_g+h_i-z)$ is the vertical distance of z away from the point P_2 , h_i is the height of the internal vegetation layer (so-called penetration height) and k_2 is von Karman's constant. β is a constant correction (approximately 1.0).

$$u = u_{P_2} \cosh\left(\frac{1}{2} \sqrt{2a\delta C_D}\right) \quad (10)$$

where u_{P_2} is the velocity of flow at a point P_2 , given by Eq. (8).

3.3. Underflow region with non-vegetated layer: Zones 3 and 4

There is no vegetation in the flow layer between P_3 to P_5 , or from the bottom of suspended vegetation to the channel bed. The governing equation in the streamwise direction becomes:

$$\frac{d\tau}{dz} = \rho gS_o \quad (11)$$

where τ is the turbulent shear stress. By applying Eq. (11) for the flow

below the suspended vegetation, where a maximum velocity is assumed to occur at $z = h_m$, i.e., at point P_4 in Fig. 3, It follows:

$$\tau - \tau_m = \rho gS_o(z - h_m) \quad (12)$$

where τ_m is the shear stress at $z = h_m$. Note that Eq. (12) works for $z \geq h_m$ (zone 3) and $z \leq h_m$ (zone 4). As a result, the different velocity profiles are obtained and explained below.

3.3.1. Velocity profile near vegetation layer: Zone 3 (P_3 to P_4)

According to the Prandtl-mixing-length hypothesis of turbulent flow, turbulent shear stress τ is described by:

$$\tau(z) = \rho l_3^2 \frac{\partial u}{\partial z} \left| \frac{\partial u}{\partial z} \right| \quad (13)$$

where l_3 is the mixing length in zone 3.

Similarly, l_3 is linearly related to d , and assume $l_3 = k_3 \delta$ where k_3 is a von Karman type of constant affected by the vegetation in vegetated flow or sediment concentration in the flow with sediment loads. Since the maximum velocity occurs at $z = h_m$, it follows that $\tau(z = h_m) = \tau_m = 0$. Inserting Eq. (13) into Eq. (12) generates the velocity gradient in layer three as follows:

$$\frac{du}{dz} = -\frac{\sqrt{gS_o(z - h_m)}}{k_3 \delta} \quad (14)$$

After the conversion of variables, Eq. (14) will have the following solution:

$$u - u_m = \frac{2u_{*3}}{k_3} \left[\sqrt{\frac{z - h_m}{h_g + h_i - h_m}} - \operatorname{arctanh}\left(\sqrt{\frac{z - h_m}{h_g + h_i - h_m}}\right) \right] \quad (15)$$

where $u_{*3} = \sqrt{gS_o(h_g + h_i - h_m)}$ and u_m is the maximum velocity at point P_4 , given by:

$$u_m = u_{P_3} + \frac{2u_{*3}}{k_3} \left[\operatorname{arctanh}\left(\sqrt{\frac{h_g - h_m}{h_g + h_i - h_m}}\right) - \sqrt{\frac{h_g - h_m}{h_g + h_i - h_m}} \right] \quad (16)$$

where u_{P_3} is the velocity of flow at a point P_3 .

3.3.2. Velocity profile near-bed layer: Zone 4 (P_4 to P_5)

In zone 4, turbulent shear stress τ is rewritten as:

$$\tau(z) = \rho l_4^2 \frac{\partial u}{\partial z} \left| \frac{\partial u}{\partial z} \right| \quad (17)$$

where l_4 is the mixing length in zone 4 and is assumed to be linearly related to z as $l_4 = k_4 z$, and k_4 is a von-Karman-type of constant in zone 4.

Similarly, by substituting Eq. (17) in Eq. (12), one can obtain:

$$\frac{du}{dz} = \frac{\sqrt{gS_o(h_m - z)}}{k_4 z} \quad (18)$$

Considering the boundary condition of $du/dz = 0$ at $z = h_m$, one can obtain the solution for u in Eq. (18) as:

$$u - u_m = \frac{2u_{*4}}{k_4} \left[\sqrt{1 - \frac{z}{h_m}} - \operatorname{arctanh}\left(1 - \frac{z}{h_m}\right) \right] \quad (19)$$

where $u_{*4} = \sqrt{gS_o h_m}$ and u_m is the maximum velocity at point P_4 , given by Eq. (16). Note, a detailed derivation of Eq. (15) for Zone 3, including all intermediate steps and mathematical transformations, is provided in Appendix A for clarity. The derivations of Eqs. (16) and (19) follow a similar approach and are briefly outlined for reference.

4. Critical parameters determination and model application

Parameters are needed to apply for the above-suggested velocity model:

(1) The Drag coefficient (C_D) was estimated using Eq. (2) for the Huai et al. (2012) model. The discussion of the C_D using different models suggests that the model presented by Huai et al. (2012) based on the regression analysis of Plew (2011) data works better than the other models, as shown in Fig. 2. In the new proposed model, C_D was estimated using Eq. (20).

$$C_D = 0.204 + 2.764\sqrt{ah_c} - \frac{0.194}{51.776\sqrt{ah_c} - 21.727} - 0.751\frac{h_c}{H} \quad (20)$$

Since the measured C_D from the Plew paper was based on the force balance, the above-fitted equation of C_D is to ensure that the velocity fits well with the measured data in the interior vegetation layer of Zone 1 (P_1 to P_2). The model for C_D using Eq. (20) was plotted against the measured value of C_D in Fig. 4. The coefficient of regression R^2 for the proposed C_D model of Eq. (20) is 0.96. The Root Mean Square Error (RMSE) between the experimental drag coefficients and the model-predicted drag coefficients is 0.124.

(2) Mixing length constants (k_2 , k_3 and k_4) are estimated for different zones of vegetation in the shear layer. In general, the conventional Karman's constant is a value of approximately 0.41, which describes the logarithmic velocity profile of a turbulent fluid flow close to a barrier with a no-slip condition. In the vegetated flow, k_2 is a proportionality constant for flow that is slowed by hindrances (like vegetation). We discovered a strong correlation between vegetation density and k_2 .

$$k_2 = 0.50(ah_c) + 0.15 \quad (21)$$

where ah_c is the variable identifying the effect of density and height of the suspended vegetation and the correlation coefficient R^2 of the proposed model for the k_2 constant is 0.97. This correlation was obtained through regression analysis across multiple test cases and produced a high coefficient of determination ($R^2 = 0.97$), indicating strong predictive reliability. The Root Mean Square Error (RMSE) between the experimental mixing length constants (k_2) and the model-predicted k_2 is 0.0038. This supports the conclusion that k_2 can be effectively parameterized in terms of vegetation structural properties, enhancing the model's applicability in varied vegetation configurations.

Meanwhile, k_3 and k_4 are the mixing length constant in zones 3 and 4 where no vegetation exists. It is expected that k_4 should be approximately 0.4 and k_3 be less than k_4 . Our study shows that k_3 and k_4 are less sensitive to vegetation density but k_2 is affected by the vegetation, as expected, because k_2 is a region closer to the suspended vegetation. As a result, the mathematical solution for the suggested model can be obtained with a constant value for both $k_3 = 0.3$ and $k_4 = 0.4$.

To provide a theoretical foundation, it is important to recognize that

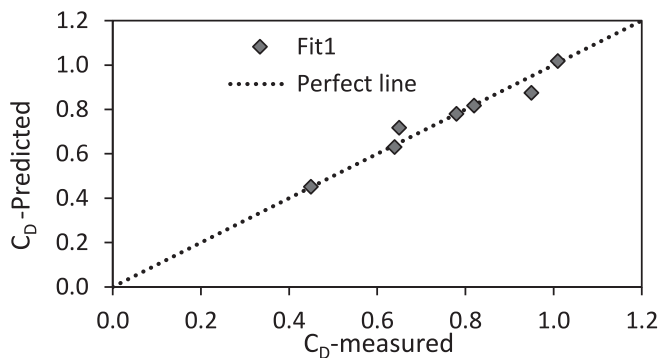


Fig. 4. This plot illustrates how well the fit equation (C_D -predicted) and the C_D estimated using Eq. (20) C_D -measured fit each other.

the variation in the mixing length constants is consistent with the Prandtl mixing-length theory, which has been adapted for heterogeneous turbulent flows. In such flows, the mixing length is known to vary with local conditions, including roughness, shear intensity, and drag distribution. The division of the flow into distinct zones, each characterized by a specific k -value, reflects the layered structure of vegetated open-channel flows, where each region is dominated by different turbulence production and dissipation mechanisms. Accordingly, the proposed parameterization of k_2 , k_3 and k_4 is not only supported by empirical correlations but also grounded in established theoretical principles of stratified turbulent flow. This dual basis enhances the physical credibility and applicability of the model across varying vegetation configurations.

(3) A shear layer that extends from the top of the vegetated layer into the non-vegetation layer is visible in the velocity profiles. The height between the bottom of the vegetation (point P_3) and the point at which longitudinal velocity in the vegetation layer starts to change suddenly (point P_2) was used to determine the depth of penetration of the shear layer into the suspended vegetation or h_i . Huai et al. (2012) showed that h_i is nearly inversely related to the vegetation drag force coefficient, demonstrating that vegetation can prevent shear vortices from developing. The penetration depth (h_i) can be obtained using Eq. (22) as a function of C_D and the flow depth H :

$$h_i = \frac{0.12H}{C_D} \quad (22)$$

The average relative error of the approximation of h_i was found to be 11.5 %. Conversely, a relationship of submerged vegetation vortex penetration ($\approx 0.25(C_D a)^{-1}$) that Ghisalberti and Nepf (2009) found to be inversely proportional to C_D was examined and found to be inapplicable in this model, which is comparable to the claim made by Huai et al. (2012).

(4) The vertical distance from the channel bed to the point P_4 , where the highest velocity occurs, is known as the position of maximum velocity (h_m). The upper vegetated zone and the channel bed at the bottom have an impact on the location of maximum velocity. The underflows in zones 3 and 4 are similar to the open channel flow beneath the ice cover because the suspended vegetation, which is more like a porous block, has a far higher flow resistance to the underflow than the channel substrate (bed). Parthasarathy and Muste (1994) stated that the underflow in a channel with ice cover is a form of asymmetric turbulent flow with heterogeneous roughness, i.e., a flow under different boundaries of roughness on the top and bottom. Guo et al. (2017) provides an analytical solution for the velocity profile in their investigation of asymmetric turbulent channel flow with applications in ice-covered rivers, and they recommended that the location of maximum velocity from the bottom bed is as:

$$\eta_m = \frac{2}{1 + \lambda^2} \quad (23)$$

where $\eta_m = h_m / (\frac{h_g}{2})$ and λ is the ratio of frictional shear velocity between the upper boundary (vegetation roughness) to the bed of the channel, as given by:

$$\lambda = \frac{u_{*v}}{u_{*b}} = \sqrt{\frac{\tau_v}{\tau_b}} \quad (24)$$

where τ_v and τ_b are the shear stress at the bottom of vegetation and the channel bed, respectively. Since $\tau_b = \rho u_{*b}^2$, u_{*b} can be approximately expressed as:

$$u_{*b} = \sqrt{g S_0 h_g} \quad (25)$$

From Eq. (9), τ_v can be obtained as follows:

$$\tau_v = \left(\rho k_2 \sqrt{\delta} \frac{du}{dz} \right) \Big|_{z=h_g} = \rho u_{*v}^2 \quad (26)$$

Eq. (27) can be obtained by applying u of Eq. (10) to Eq. (25), which will provide u_v , and then combining it with Eq. (24).

$$\frac{h_m}{h_g} = \frac{1}{\cosh^2\left(\frac{1}{k_2} \sqrt{2a\delta C_D h_i}\right)} \quad (27)$$

Thus, Eq. (27) gives an estimated value of h_m . Notably, Eq. (27) shows that h_m depends on C_D , flow depth, and vegetation height and density, suggesting the potential to relate maximum and surface velocities.

The flow chart in Fig. 5 can be used to demonstrate how the suggested model is applied. All the zones, along with the parameters and mathematical model equations used to predict the velocity distributions, are provided in a manner that allows users to apply them by following the steps shown in Fig. 5. Applying the model is simple, and Plew's (2011) experimental datasets are used to test the zone-wise technique.

5. Results

Fig. 6 validates the vertical distribution of streamwise velocities for runs B2, B5, B9, and B13. The study by Huai et al. (2012) tested these test cases. In Fig. 6(a–d), the proposed model and the model by Huai et al. (2012) are thus compared with the experimental results of the vertical streamwise velocity distribution. The flow depth ($H = 0.2$ m) and projected area per unit volume ($\alpha = 1.272 \text{ m}^{-1}$) are constant in all these test cases used in the study of Huai et al. (2012). However, B2 and B13 have different distances between the vegetation and bed; B2 has the smallest distance (0.025 m) between the suspended vegetation and bed, while B13 has the most (0.1 m). In essence, the vegetation length decreases from test case B2 to test case B13. As a result, B2 has the strongest interaction between the bottom boundary layer and the vegetation shear layer, and zones 3 and 4 are smaller. To quantitatively compare the difference between the predicted and the measured velocities, the root mean square error (RMSE) was computed as:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (u_{exp}(z) - u_p(z))^2} \quad (28)$$

where N is the number of measurements in the vertical direction, $u_{exp}(z)$ and $u_p(z)$ are the measured and predicted velocities, respectively.

The projected outcomes of the suggested models and the experimental data are in good agreement, as shown in Fig. 6. The outcomes of the Huai et al. (2012) model likewise show good performance for the B2 test scenarios with $RMSE/U_o$ value of 5.58 % in comparison to the $RMSE$ value of 5.79 % for the proposed model (Fig. 6a). As suggested in their paper, k_2 , k_3 , and k_4 were used in their model as the average values of the range, or around 0.28, 0.3, and 0.4, respectively. The estimated velocities distribution for B5, B9, and B13 for the proposed model ($RMSE/U_o \approx B5 = 7.55\%$; $B9 = 11.67\%$; and $B13 = 17.17\%$) perform better than those using Huai et al. (2012) model ($RMSE/U_o \approx B5 = 25.36\%$; $B9 = 18.02\%$; and $B13 = 28.77\%$). Further, our model performs better in the non-vegetated area and catches the inflection, maximum velocity, and bottom boundary layer in every scenario. For instance, the percentage error $((u_{m,observed} - u_{m,predicted})/u_{m,observed})$ for the estimated u_m value for the proposed model is in the range of 1–4 % compared to the Huai et al. (2012) model with a percentage error range of 11–25 %. However, for the test run B13, there are minor differences between the predicted and observed values near the water surface see Fig. 6(d). The possibility that the turbulent shear layer, disregarded in this layer, underestimates computed velocity, or the surface wave distribution has introduced some abnormalities in the experimental data collection, making these plausible.

The vertical distribution of streamwise velocities for the test run, with different projected vegetation areas per unit volume and density, is measured and analytically estimated in Fig. 7a–c. Out of all the test runs, B14 and B15 had the lowest projected area per unit volume, having less vegetation per unit area than B12, B2, B5, B9, and B13. In Huai et al. (2012) analysis, the B12, B14, and B15 test runs were not tested. The following suggested semi-analytical model results outperform the Huai et al. (2012) model for different vegetation densities, which must be incorporated into the previously modeled solutions. For the B12, B14, and B15 test runs, the $RMSE/U_o$ value is 11.56 %, 11.00 %, and 7.83 %, respectively, for the suggested model. In contrast, the Huai et al. (2012) model's $RMSE/U_o$ value for the B12, B14, and B15 test runs is 17.99 %, 31.47 %, and 24.72 %, respectively. The point of inflection and

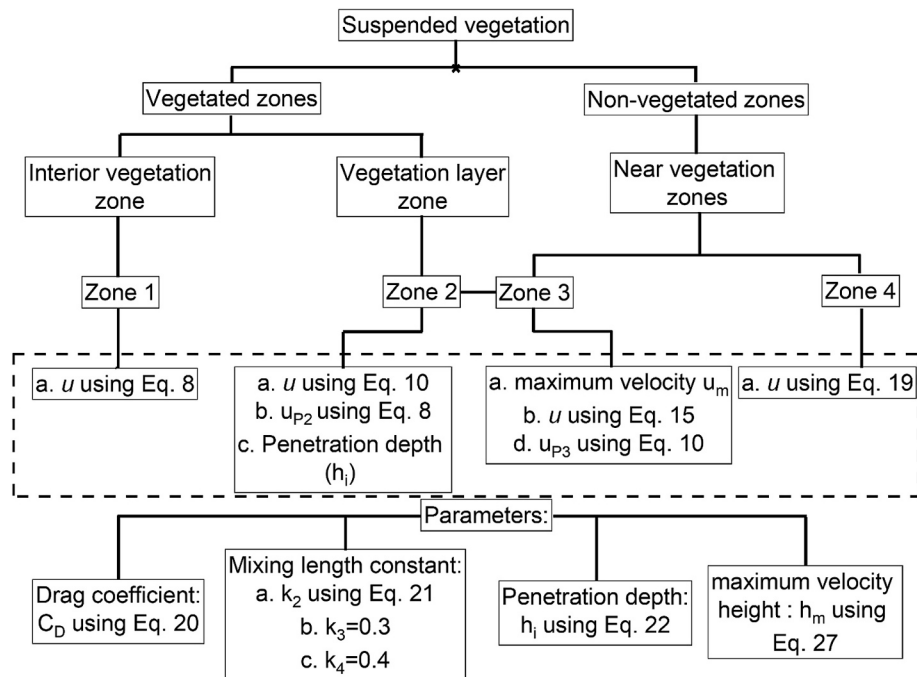


Fig. 5. The flow chart explains the model application for each zone. The derived equations and parameters for estimating velocity in suspended vegetation are given for each zone.

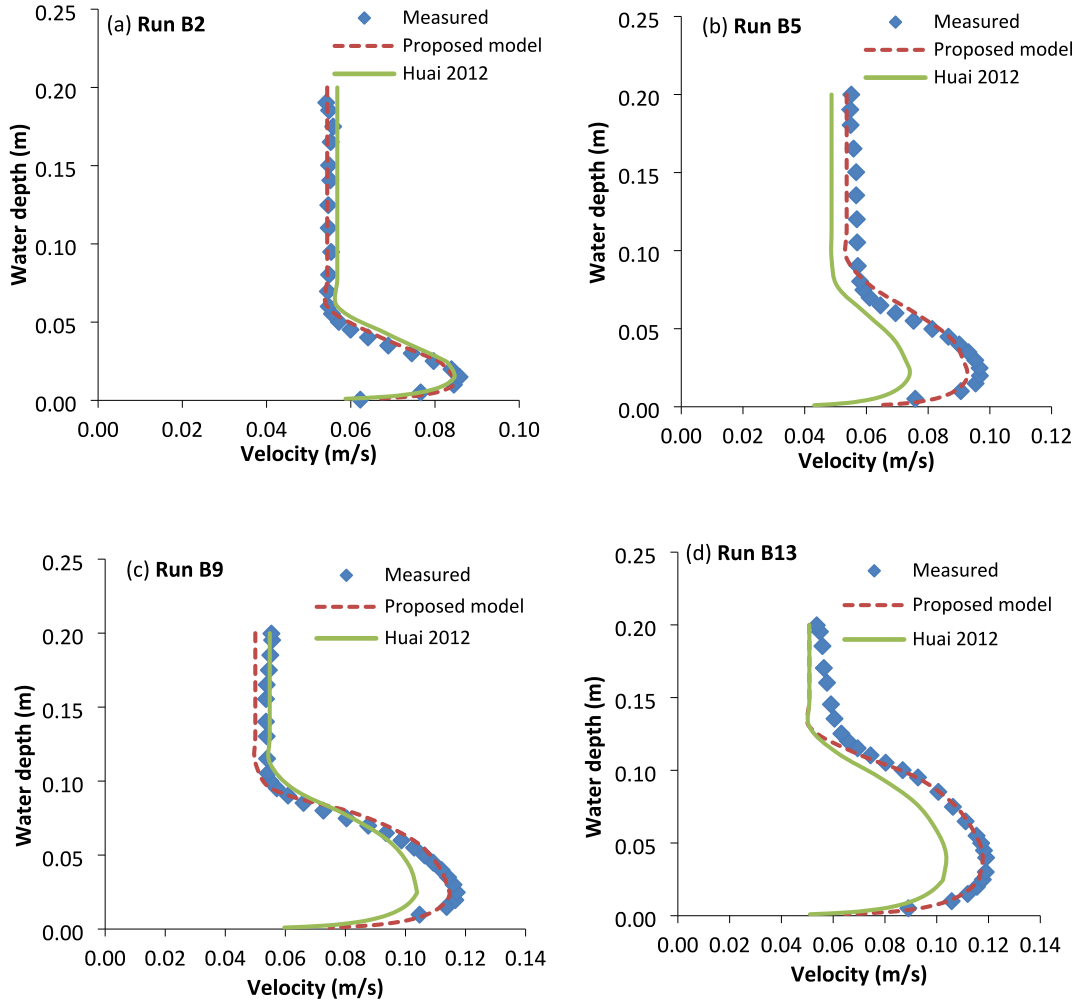


Fig. 6. The comparison of the experimental data of the vertical distribution of streamwise velocities with the proposed semi-analytical model.

maximum velocity are well captured in the transition layer for the proposed model in Fig. 7. The percentage error $((u_{m,observed} - u_{m,predicted})/u_{m,observed})$ for the estimated u_m by proposed model for B12, B14 and B15 are 3 %, 2 % and -0.4 %, respectively. However, for the Huai et al. (2012) model, the range of percentage error for the estimated u_m is 20–25 %. Again, the proposed model underestimates slightly in the region adjacent to the water surface. However, the $RMSE/U_o$ value for the constant velocity region of Zone 1 using the proposed model is in the range of 0.92–16.67 %, with the lowest percentage for B2, B5, B9, and B15. For the Huai et al. (2012) model, $RMSE/U_o$ value for the constant velocity region of Zone 1 is in the range of 3.36–25.71 %. In summary, the proposed model has outperformed the prior model for the various densities, suspended vegetation gaps, and their configurations.

6. Discussions

The data from Plew (2011) are used to verify the proposed model. It should be noted that only the vertical profile of velocities in the fully developed flow area was measured in these investigations. The suggested model shows good predictions with data (Figs. 6 & 7) for a wide vegetation condition ($a = 0.4777$ to 1.908 m^{-1}) with $RMSE/U_o = 5.8\sim 17$ %. These results demonstrate that the proposed model can correctly predict the vertical velocity profiles in the situation of suspended vegetation. Table 2 also illustrates the correlation coefficients for each test case. This analysis demonstrates that the proposed model shows better agreement with the measured data than the Huai et al.

(2012) model across all cases. The current results show that the zone-wise proposed model can be used to predict the vertical distribution of the velocity in suspended vegetation using five parameters as drag coefficient (C_D), mixing length constants (k_2 , k_3 and k_4), penetration depth (h_i) and the position of the maximum velocity (h_m). In comparison to the previous study of Huai et al. (2012), mixing length constants range: $0.2 < k_2 < 0.4$; $0.12 < k_3 < 0.35$; $0.33 < k_4 < 0.58$ are re-defined in the present study. According to the analysis that follows, the effect of vegetation on k remains constant the further the layer is from the vegetation. It is commonly known that in open-channel flow, k (von-Karman) is about 0.41 in clear water. It is established in this study that k_4 will be roughly 0.4 and k_3 will be less than k_4 . According to our study, k_2 is impacted by the vegetation, as would be expected given that it is a location that is closer to the suspended vegetation, whereas k_3 and k_4 are less responsive to plant density.

Furthermore, the inconsistent prediction of zone 1 with the maximum drag region and constant velocity is a problem with the previous model. Some inconsistency in velocity prediction in zone 1 of B9 & B13 can be improved if their C_D is slightly reduced to 0.95 from 1.01 (B9) and 0.85 from 0.95 (B13). Thus, the new model of C_D (Eq. (20)) should be used rather than Eq. (2) in the previous analysis. The predicted profile matches the underflow region with non-vegetated layers of Zone 3 and 4 with the least error in general. It is to be noted that the models of Huai et al. (2012), also used by Gui et al. (2024), use logarithmic function to predict the profile. However, in the proposed model, the inverse hyperbolic tangent function is used, which predicts with the least RMSE and overall percentage error for the test runs considered in

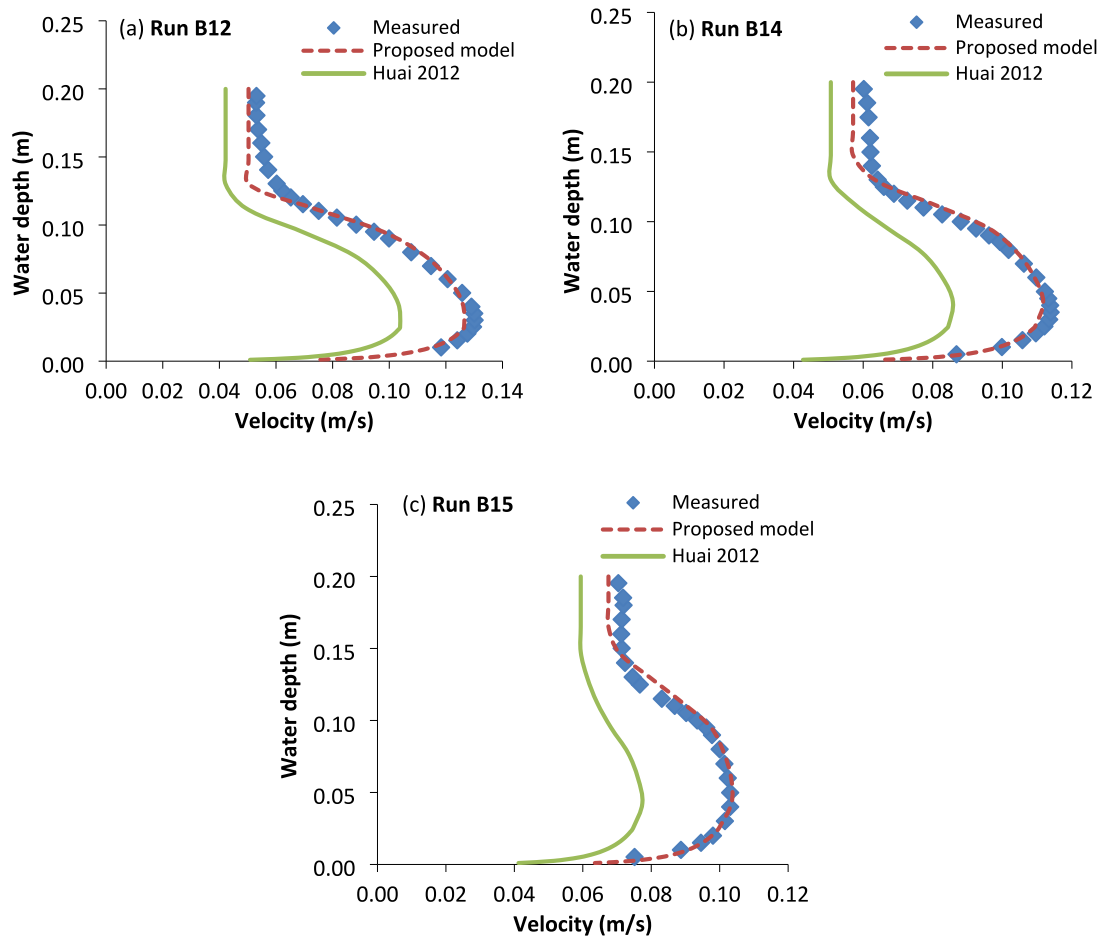


Fig. 7. The comparison of the experimental data of the vertical distribution of streamwise velocities with the proposed semi-analytical model (for the data not mentioned in the study of [Huai et al., 2012](#)).

Table 2

Correlation coefficients for test runs B2–B15 from [Plew \(2011\)](#), comparing the proposed model with the model by [Huai et al. \(2012\)](#).

Test cases	Proposed model	Huai et al. (2012)
Run B2	0.7820	0.6887
Run B5	0.8372	0.4016
Run B9	0.8738	0.5615
Run B13	0.7000	0.4578
Run B12	0.7718	0.3126
Run B14	0.9257	0.5512
Run B15	0.8080	0.3471

this analysis.

[Fig. 8\(a–b\)](#) shows the percentage error $((u_{\text{observed}} - u_{\text{predicted}})/u_{\text{observed}})$ of predicted average velocity within and beneath the vegetation using the proposed and [Huai et al. \(2012\)](#) models. Two distinct areas of suspended vegetation—the vegetated and non-vegetation zones—are of special importance to researchers and field engineers since the model was developed for distinct flow zones with averaged values of U_c and U_b . Comparatively, [Fig. 8\(a\)](#) shows that the proposed model has fewer percentage errors for predicting average velocity within the suspended vegetation, except for B2 for U_c . For the highest density suspended vegetation test case B12, the proposed model has a percentage error of 15 %, which is less than the [Huai et al. \(2012\)](#) model. Moreover, [Fig. 8\(b\)](#) consistently demonstrates that the suggested model performs well in the region below, with an average velocity U_b percentage error of less than 5 %.

There are two ways to talk about some of the modeling drawbacks.

There are still some inconsistencies in the prediction of zone 1 with the maximum drag region in the overall modeling. The drag coefficient value for a larger variety of channels and suspended vegetation patterns should be empirically explored, even though the suggested model has the lowest RMSE and percentage error in zone 1 region. Moreover, the study of surface wave distribution in the experimental data of suspended vegetation needs more attention as this would affect the effective depth. Second, this study only looked at the presence of rigid, cylindrical suspended vegetation. Testing the model in the flexible suspended vegetation is necessary.

Furthermore, only experimental configurations were used with the current model. Applying the recently suggested model to field datasets in suspended vegetation flows and the vertical distribution of streamwise velocity in riverine flows is necessary. Moreover, additional field data and experimental research are advised for future practical uses. Finally, it can be concluded that the new mathematical model can predict velocity distribution in suspended vegetation. Modelling demonstrates the significance of including mixing length at the vegetation shear layer as a function of flow depth and density to predict the inflection most accurately.

7. Conclusions

A zone-wise semi-analytical model has been introduced for open channel flow through suspended rigid vegetation. Based on the given density of suspended vegetation, this model may offer a vertical profile of streamwise velocities, which are often needed to evaluate design issues in artificial wetland habitats, especially those with solute and

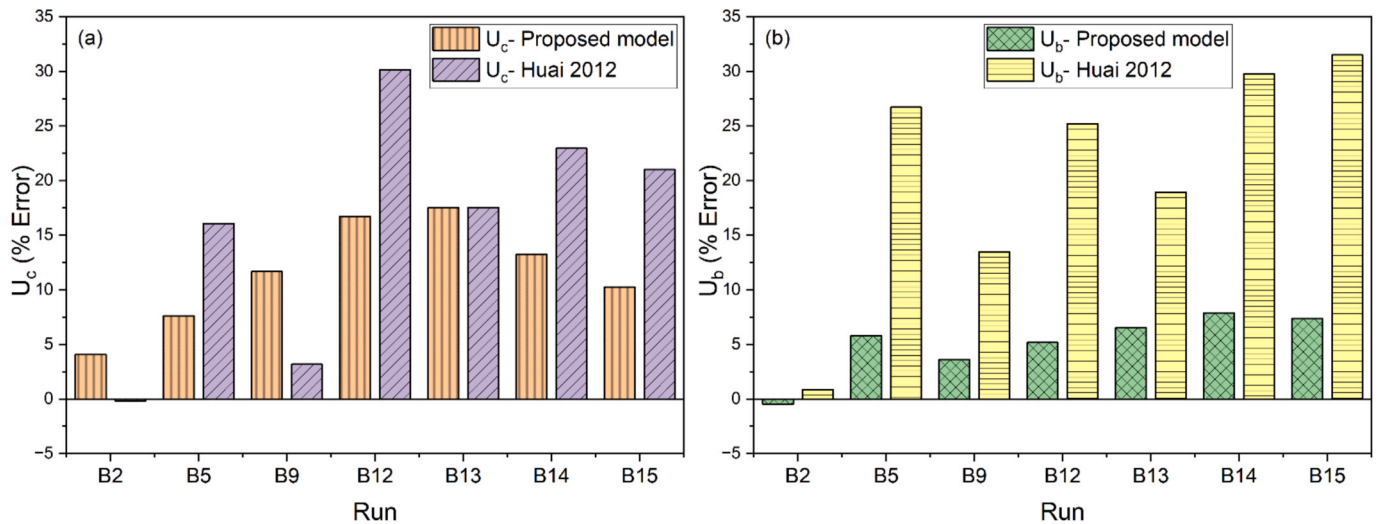


Fig. 8. The comparison of the percentage error for the averaged values of U_c and U_b for the semi-analytical proposed and Huai et al. (2012) model.

sediment transport, in contrast to widely used 1D analytical techniques. In contrast to numerical simulation techniques, the semi-analytical model is easy to use and straightforward. Based on the blockage ratio, canopy density and drag, and bed friction values, the suggested semi-analytical model accurately predicts the vertical distribution of streamwise velocities and provides a maximum velocity (h_m) location. The model shows how the vegetation density and blockage ratio influence vegetation shear layers in individual zones. It is seen that the increasing vegetation density produces higher bed friction because velocities under the vegetation are increased. Thus, mixing length constants are derived for the near vegetation zone as the linear function of the density and vegetation height. However, the modeling of mixing length in the non-vegetated zone suggests that a constant value for near vegetation and bed zone is sufficient to predict the velocity.

The analysis was limited to rigid floating vegetation under varying submergence heights. Future studies should incorporate flexible floating vegetation to better represent interface mixing influenced by dynamic plant motion. Additionally, the proposed model was validated only with laboratory data. Broader validation using field measurements and more complex channel configurations, including diverse vegetation arrangements, is essential to enhance the model's reliability for practical applications such as floodplain restoration and pollutant dispersion in urban and natural waterways. Further research should also investigate the role of marginal vegetation in drainage systems, canals, and reservoirs, considering both hydraulic and ecological impacts. Moreover, since measuring maximum velocity and its corresponding height (h_m) in the field is challenging, future work should explore the development of empirical correlations between easily measurable surface velocity and maximum flow velocity. This would significantly improve the model's practical usability for field engineers and monitoring agencies.

CRediT authorship contribution statement

Xiaonan Tang: Writing – review & editing, Supervision, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Prateek**

Appendix A

Here is the derivation of Eq. (15) for Zone 3. Based on Eq. (14), we rewrite it as:

$$\frac{du}{dz} = -\frac{\sqrt{gS_0(z-h_m)}}{k_3\delta} \quad (A1)$$

Kumar Singh: Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **Hamidreza Rahimi:** Visualization, Validation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Declarations ethics approval and consent to participate

This article contains no studies with human participants or animals performed by authors.

Consent to publish

The authors declare no conflict of interest.

Institutional review board statement

Not applicable.

where $\delta = h_g + h_i - z$. Let $y^2 = z - h_m$, and $M = \frac{\sqrt{gS_0}}{k_3}$. Then using $\frac{du}{dz} = \frac{du}{dy} \frac{dy}{dz} = \frac{1}{2y} \frac{du}{dy}$, and let $h_z = h_g + h_i - h_m$. Eq. (A1) becomes:

$$\frac{1}{M} \frac{du}{dy} = -\frac{2h_z}{(\sqrt{h_z})^2 - y^2} \quad (\text{A2})$$

According to 'Mathematical Handbook of Formulas and Tables' (5th edition) by Spiegel et al. (2018, p76): when $x^2 > a^2$:

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left(\frac{x-a}{x+a} \right) + c = -\frac{1}{a} \operatorname{arccoth} \left(\frac{x}{a} \right) + c \quad (\text{A3})$$

where c is integration constant, a is a constant.

Thus, Eq. (A2) can be written as:

$$\frac{u}{M} = 2y + \sqrt{h_z} \ln \left(\frac{\sqrt{h_z} - y}{\sqrt{h_z} + y} \right) + c \quad (\text{A4})$$

Using the boundary conditions as follows:

If $z = h_m$, $y = 0$, and $u = u_m$ (the maximum velocity at $z = h_m$), thus $c = u_m/M$. Replacing c into Eq. (A4) yields:

$$\frac{u - u_m}{M} = 2y - \sqrt{h_z} \ln \left(\frac{\sqrt{h_z} + y}{\sqrt{h_z} - y} \right) = 2y - 2\sqrt{h_z} \operatorname{arctanh} \left(\frac{y}{\sqrt{h_z}} \right) \quad (\text{A5})$$

Replacing M and h_z in Eq. (A5) and rearranging the resulting equation yields:

$$u - u_m = \frac{2u_{*3}}{k_3} \left[\sqrt{\frac{z - h_m}{h_g + h_i - h_m}} - \operatorname{arctanh} \left(\sqrt{\frac{z - h_m}{h_g + h_i - h_m}} \right) \right] \quad (\text{A6})$$

where $u_{*3} = \sqrt{gS_0(h_g + h_i - h_m)}$ and u_m is the maximum velocity at point P4 (i.e. $z = h_g$), given by:

$$u_m = u_{p3} + \frac{2u_{*3}}{k_3} \left[\operatorname{arctanh} \left(\sqrt{\frac{h_g - h_m}{h_g + h_i - h_m}} \right) - \sqrt{\frac{h_g - h_m}{h_g + h_i - h_m}} \right] \quad (\text{A7})$$

Similarly, Eq. (19) can be obtained following the same settings ($y^2 = h_m - z$).

Data availability

The data presented in this study are available on request from the corresponding author. Furthermore, the authors would also like to sincerely thank all the past researchers who gave valuable experimental datasets.

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